AP Calculus Name________________________

Chapter 7 Review

1. [No Calculator] Given $v(t) = \sqrt{4-t}$, $0 \le t \le 4$

$(\sqrt{4-t})^2 = (0)^2$ $4-t=0$ -4 -4 $-t=-4$ $t=4$

a. Determine when the particle is moving to the right, to the left, and stopped.

right, $v(t) > 0$ $\sqrt{4-t} > 0$ $0 \le t < 4$

left, $v(t) < 0$ $\sqrt{4-t} < 0$ never

stopped $v(t) = 0$ $\sqrt{4-t} = 0$ $t = 4$

b. Find the particle's displacement for the given time interval.

$$\int_0^4 (4-t)^{1/2}\,dt = -\frac{(4-t)^{3/2}}{3/2}\Bigg|_0^4 = -\frac{(4-4)^{3/2}}{3/2} - -\frac{(4-0)^{3/2}}{3/2} = 0 + \frac{2 \cdot 4^{3/2}}{3} = 0 + 16/3 = 16/3$$

c. If s(0) = 3, what is the particle's final position?

$$s(0) + \int_0^4 (4-t)^{1/2}\,dt = 3 + \frac{16}{3} = \frac{9}{3} + \frac{16}{3} = \frac{25}{3}$$

2. [Calculator] A particle travels with velocity $v(t) = (t-2)\sin t$ m/sec. for $0 \le t \le 4$

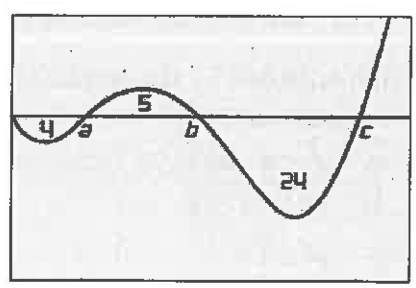
a. What is the particle's displacement?

$$\int_0^4 (t-2)\sin t\,dt = -1.449515 \text{ meters}$$

b. What is the total distance traveled?

$$\int_0^4 |(t-2)\sin t|\,dt = 1.91411 \text{ meters}$$

[No Calculator] In exercises 3-7, a particle moves along the x-axis (units in cm). Its initial position at $t = 0$ sec. is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the areas of the enclosed regions.



3. What is the particle's displacement between $t = 0$ and $t = c$?

$-4+5-24 = -23$ cm

4. What is the total distance traveled by the particle in the same time period, from (0, c)?

$4+5+24 = 33$ cm

5. Give the positions of the particle at times a, b, and c.

start (0,15) $t=a$ $x=11$ $t=c$ $x=-8$

$t=b$ $x=16$

6. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$?

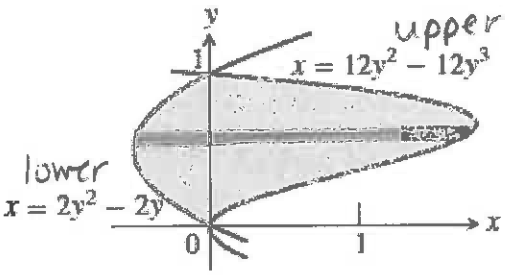
"b" a max of acceleration will occur when $v(t)$ is steepest (and positive)

7. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$?

"c" for same reasons above

Find the area between the curves. [No Calculator]

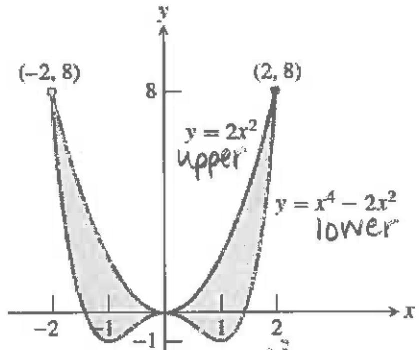
8.



$$\int_0^1 12y^2-12y^3-(2y^2-2y)\,dy = \int_0^1 -12y^3-10y^2+2y\,dy$$

$$= -\frac{12y^4}{4}+\frac{10y^3}{3}+y^2\Big|_0^1 = -3+\frac{10}{3}+1-(0-0+0)$$
$$= -\frac{9}{3}+\frac{10}{3}+\frac{3}{3} = \boxed{4/3}$$

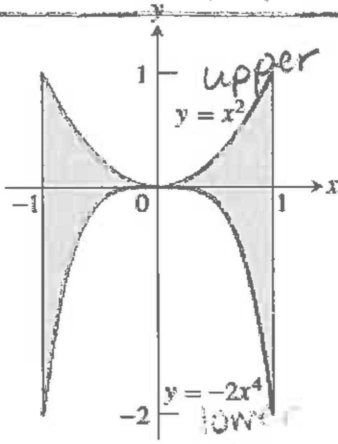
9.



$$\int_{-2}^{2} 2x^2-(x^4-2x^2)\,dx = \int_{-2}^{2} -x^4+4x^2\,dx$$

$$= -\frac{x^5}{5}+\frac{4x^3}{3}\Big|_{-2}^{2} = -\frac{(2)^5}{5}+\frac{4(8)}{3}-\left(-\frac{(-2)^5}{5}+\frac{4(-8)}{3}\right)$$
$$= -\frac{32}{5}+\frac{32}{3}-\frac{32}{5}+\frac{32}{3} = -\frac{64}{5}+\frac{64}{3} = -\frac{192}{15}+\frac{320}{15} = \boxed{\frac{128}{15}}$$

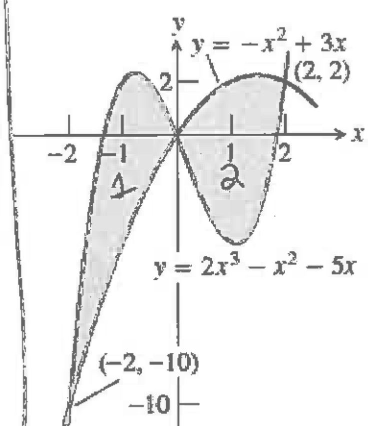
10.



$$\int_{-1}^{1} x^2-(-2x^4)\,dx = \int_{-1}^{1} 2x^4+x^2\,dx$$

$$= \frac{2x^5}{5}+\frac{x^3}{3}\Big|_{-1}^{1} = \frac{2}{5}+\frac{1}{3}-\left(-\frac{2}{5}-\frac{1}{3}\right)$$
$$= \frac{4}{5}+\frac{2}{3} = \frac{12}{15}+\frac{10}{15} = \boxed{\frac{22}{15}}$$

11. Find the total area.



upper & lower change!

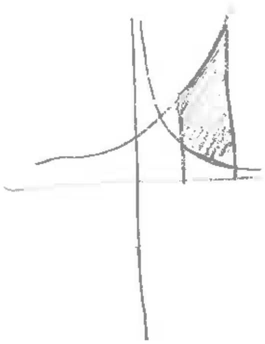
$$\int_{-2}^{0} [2x^3-\cancel{x^2}-5x-(-\cancel{x^2}+3x)]\,dx + \int_{0}^{2} [-\cancel{x^2}+3x-(2x^3-\cancel{x^2}-5x)]\,dx$$

$$\int_{-2}^{0} 2x^3-8x\,dx + \int_{0}^{2} 8x-2x^3\,dx = \frac{2x^4}{4}-4x^2\Big|_{-2}^{0} + 4x^2-\frac{2x^4}{4}\Big|_{0}^{2}$$

$$= 0-\left(\frac{2(-2)^4}{4}-4(-2)^2\right) + 4(2)^2-\frac{2(2)^4}{4}-0$$
$$-(8-16) + 16-8 = --8+8 = 8+8 = \boxed{16}$$

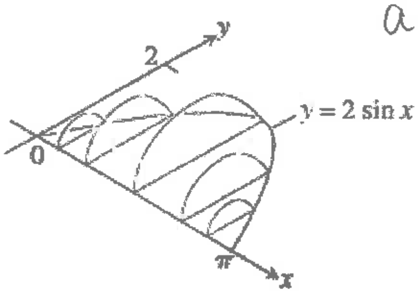
12. Let f and g be the functions given by $f(x)=e^x$ and $g(x)=\frac{1}{x}$. Which of the following gives the area of the region enclosed by the graphs of f and g between $x=1$ and $x=2$.

a. $e^2-e-\ln 2$
b. $\ln 2-e^2+e$
c. $e^2-\frac{1}{2}$
d. $e^2-e-\frac{1}{2}$
e. $\frac{1}{2}-\ln 2$



$$\int_1^2 e^x-\frac{1}{x}\,dx = e^x-\ln x\Big|_1^2$$
$$= e^2-\ln 2-(e^1-\ln 1)$$
$$= e^2-\ln 2-e+0$$
$$= e^2-\ln 2-e$$

13. [Calculator] A solid has a base that lies between the x-axis and one arch of the curve $y = 2\sin x$. Each cross section cut perpendicular to the x-axis is a semicircle. Find the volume of the paperweight.



area $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi\left(\frac{2\sin x}{2}\right)^2$

$= \frac{1}{2}\pi \sin^2 x$

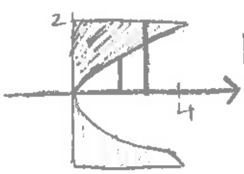
$\int_0^{\pi} \frac{1}{2}\pi \sin^2 x \, dx = \frac{1}{2}\pi \int_0^{\pi} \sin^2 x \, dy = 2.4674$

14. [No calculator] Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about

a. the x-axis

washer

(use x-values)



$R = 2$

$r = \sqrt{x}$

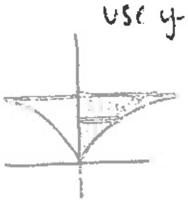
$\pi \int_0^4 2^2 - (\sqrt{x})^2 dx = \pi\left(4x - \frac{x^2}{2}\right)\Big|_0^4 = \pi\left(4 \cdot 4 - \frac{4^2}{2}\right) - 0$

$= \pi(16 - 8) = \boxed{8\pi}$

b. the y-axis

disc

use y-values



$y = \sqrt{x}$

$x = y^2$

$\pi \int_0^2 (y^2)^2 dy = \pi \int_0^2 y^4 dy = \frac{y^5}{5}\Big|_0^2 = \pi\left(\frac{2^5}{5} - \frac{0^5}{5}\right) = \pi\frac{32}{5} = \boxed{\frac{32\pi}{5}}$

c. the line $y = 2$

disc

x-values

$\pi \int_0^4 (2-\sqrt{x})^2 dx = \pi \int_0^4 (4 - 4\sqrt{x} + x) dx = \pi\left(4x - \frac{2}{3} \cdot 4x^{3/2} + \frac{x^2}{2}\right)\Big|_0^4$

$(2-\sqrt{x})(2-\sqrt{x})$

$4 - 2\sqrt{x} - 2\sqrt{x} + x$

$= \pi\left[\left(4(4) - \frac{2}{3} \cdot 4 \cdot 8 + 8\right) - (0 - 0 + 0)\right]$

$= \pi\left(16 - \frac{64}{3} + 8\right) = \pi\left(24 - \frac{64}{3}\right) = \pi\left(\frac{72}{3} - \frac{64}{3}\right)$

$\boxed{= \frac{8\pi}{3}}$

d. the line $x = 4$

washer

y-values

$y = \sqrt{x}$

$x = y^2$

$R = 4$

$r = 4 - y^2$

$\pi \int_0^2 4^2 - (4-y^2)^2 dy = \pi \int_0^2 16 - (16 - 8y^2 + y^4) dy$

$= \pi \int_0^2 8y^2 - y^4 dy = \pi\left(\frac{8y^3}{3} - \frac{y^5}{5}\right)\Big|_0^2$

$= \pi\left(\frac{64}{3} - \frac{32}{5}\right) = \pi\left(\frac{320}{15} - \frac{96}{15}\right) = \boxed{\frac{224}{15}\pi}$