KEY

P Calculus
.1 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The only way to guarantee the existence of a limit is to algebraically prove it. Describe the different ways you can <u>*investigate*</u> the existence of a limit.

2. Using words, explain what is meant by the expression $\lim_{q \to c} f(q) = T$.

The limit of f(q) as q approaches c is the value T.

3. How do you find the average speed of an object?

$$\frac{\text{displacement (distance)}}{\text{time}}$$

4. Suppose an object moves along the x-axis with it's position function given by $x(t) = 5t^2 + 7t$, where t is measured in seconds.

a) What is the average speed from $t = 2$ to $t = 4$ seconds?

$x(4) = 108 \quad x(2) = 34$

$$\frac{108 - 34}{4 - 2} = \frac{74}{2} = 37 \text{ ft/sec.}$$

b) How fast is the object moving at exactly $t = 4$ seconds?

$$\frac{x(4.001) - x(4)}{4.001 - 4} = 47.005 \text{ ft/sec}$$

$$\approx 47 \text{ ft/sec.}$$

$$\frac{x(4) - x(3.999)}{4 - 3.999} = 46.995 \text{ ft/sec}$$

5. An rover on another planet drops an object off a cliff. The object falls $y = gt^2$ m in t sec, where g is a constant. Five seconds after the object was dropped it lands 30 m below.

a) Find the value of g.

$30 = g(5)^2$

$30 = g(25)$

$g = \frac{30}{25} = 1.2$

b) Find the average speed for the fall.

$$\frac{30 \text{ ft}}{5 \text{ sec}} = 6 \text{ ft/sec.}$$

c) With what speed did the rock hit the bottom?

$t = 5$

$$\frac{y(5) - y(4.999)}{5 - 4.999} \quad 11.998 \quad \approx 12 \text{ ft/sec}$$

6. Assume $\lim_{x \to b} f(x) = 7$ and $\lim_{x \to b} g(x) = -3$.

a) $\lim_{x \to b} (f(x) + g(x)) =$

$7 + -3 = 4$

b) $\lim_{x \to b} (f(x) \cdot g(x)) = 7 \cdot -3 = \boxed{-21}$

c) $\lim_{x \to b} 4g(x) =$

$4 \cdot \lim_{x \to b} g(x) = 4 \cdot -3$

$\boxed{= -12}$

d) $\lim_{x \to b} \left(\frac{f(x)}{g(x)} \right) = \boxed{\frac{7}{-3}}$

7. When asked to evaluate the limit of a function, what should be done first? try to substitute

8. Evaluate the following limits by using direct substitution.

a) $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right)$ $= \sec \frac{7\pi}{6}$

$= \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -1.155$

b) $\lim_{x\to 4} \sqrt[3]{x+4}$ $= \sqrt[3]{4+4} = \sqrt[3]{8} = 2$

c) $\lim_{x\to ½} 3x^2(2x-1)$ $= 3(½)^2(2(½)-1)$

$3 \cdot ¼ \cdot 0 = 0$

d) $\lim_{y\to 2} \frac{y^2+5y+6}{y+2}$ $= \frac{2^2+5(2)+6}{2+2}$

$= \frac{4+10+6}{4} = \frac{20}{4} = 5$

e) $\lim_{x\to -2} (x-6)^{⅔}$ $= (-2-6)^{2/3}$

$= (-8)^{2/3}$

$= \sqrt[3]{-8}^{\,2} = -2^2 = 4$

f) $\lim_{x\to 2} \sqrt{x+3}$

$\sqrt{2+3} = \sqrt{5}$

9. Explain why you cannot use direct substitution to determine each of the following limits.

a) $\lim_{x\to -2} \sqrt{x-2}$

$\sqrt{-4}$ can't $\sqrt{-}$

b) $\lim_{x\to 0} \frac{1}{x^2}$

$\frac{1}{0}$

can't divide by 0.

c) $\lim_{x\to 0} \frac{(4+x)^2-16}{x}$

$\frac{0}{0}$

can't divide by 0.

10. If a limit does not exist, there are 3 possible reasons why. List all three possible reasons why a limit may not exist.

as $x \to c$

f(x) approaches a different number from right than left

f(x) increases or decreases without bound

f(x) oscillates between two fixed values

11. Find each limit, or explain why the limit does not exist.

a) $\lim_{x\to 2} f(x)$, if $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln x & \text{for } 2 < x \le 4 \end{cases}$

$\ln(2) = \ln 2$

$2^2 \ln(2) = 4\ln 2$

f(x) does not approach same value from left & right

b) $\lim_{x\to 2^+} f(x)$, if $f(x) = \begin{cases} 3x+1 & ,x<2 \\ \frac{5}{x+1} & ,x \ge 2 \end{cases}$ ← use this function

from right

$\frac{5}{2+1} = \boxed{\frac{5}{3}}$

c) $\lim_{x\to 1} \frac{x^2-4}{x-1}$

$\frac{(x+2)(x-2)}{x-1}$

increases & decreases w/o bound @ 1

d) $\lim_{x\to 2} \frac{x+1}{x^2-4}$

$\frac{(x+1)}{(x-2)(x+2)}$

increases & decreases w/o bound @ x=2

12. Determine whether each statement about the graph below is True or False.

a) $\lim\limits_{x\to -1^+} f(x)=1$

T

b) $\lim\limits_{x\to 2} f(x)$ does not exist

F

c) $\lim\limits_{x\to 2} f(x)=2$

F

d) $\lim\limits_{x\to 1^-} f(x)=2$

T

e) $\lim\limits_{x\to 1^+} f(x)=1$

T

f) $\lim\limits_{x\to 1} f(x)$ does not exist

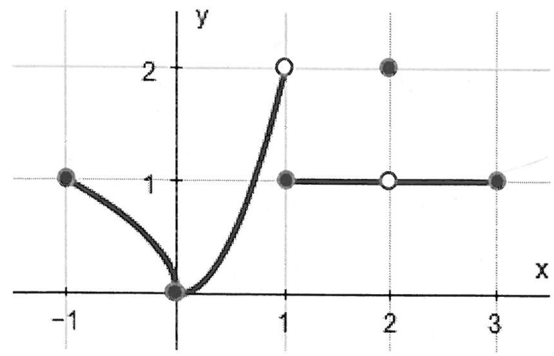
T

g) $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^-} f(x)$

T

h) $\lim\limits_{x\to c} f(x)$ exists at every c in $(-1, 1)$

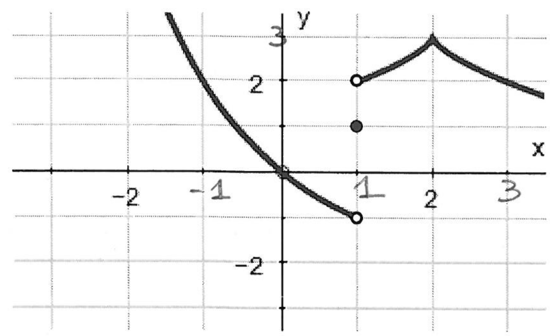
T



i) $\lim\limits_{x\to c} f(x)$ exists at every c in $(1, 3)$

F (no limit as $x \to 1$)

13. Use the graph of $f(x)$ to estimate the limits and value of the function, or explain why the limit does not exist.



a) $\lim\limits_{x\to 1^+} f(x)$

2

b) $\lim\limits_{x\to 1^-} f(x)$

-1

c) $\lim\limits_{x\to 1} f(x)$

DNE

d) $f(1)$

1

e) $\lim\limits_{x\to 2^+} f(x)$

3

f) $\lim\limits_{x\to 2^-} f(x)$

3

g) $\lim\limits_{x\to 2} f(x)$

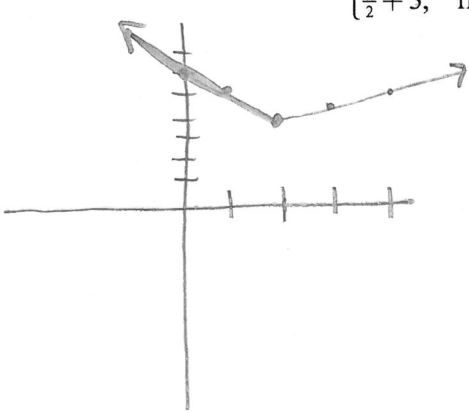
3

h) $f(2)$

3

14. For each of the following functions, (i) draw the graph, (ii) determine $\lim\limits_{x\to c^+} f(x)$ and $\lim\limits_{x\to c^-} f(x)$, and (iii) explain what the value of $\lim\limits_{x\to c} f(x)$ is or explain why it doesn't exist.

a) $c=2,\ f(x)=\begin{cases} 6-x, & \text{if } x<2 \\ 4, & \text{if } x=2 \\ \frac{x}{2}+3, & \text{if } x>2 \end{cases}$



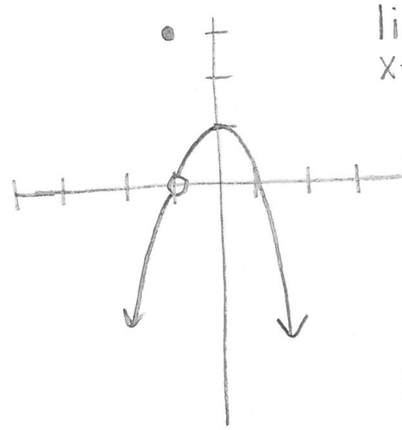
$\lim\limits_{x\to 2^+} f(x)=4$

$\lim\limits_{x\to 2^-} f(x)=4$

$\lim\limits_{x\to 2} f(x)=4$

two sided limit!

b) $c=-1,\ f(x)=\begin{cases} 1-x^2, & \text{if } x\neq -1 \\ 3, & \text{if } x=-1 \end{cases}$



$\lim\limits_{x\to -1^-} f(x)=0$

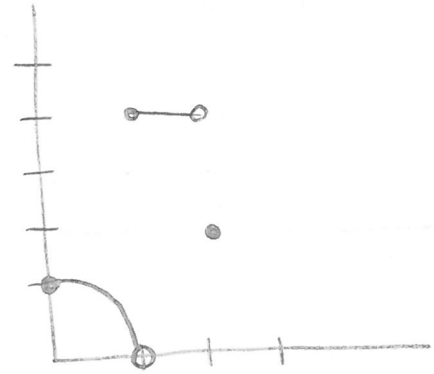
$\lim\limits_{x\to -1^+} f(x)=0$

$\lim\limits_{x\to -1} f(x)=0$

LH limit = RH limit

x	0	1
y	1	0

15. Suppose $f(x)=\begin{cases}\sqrt{1-x^2}, & \text{if } 0\le x<1\\ 3, & \text{if } 1\le x<2\\ 1, & \text{if } x=2\end{cases}$. Draw a graph of $f(x)$, then answer the following questions.



a) At what points c in the domain of f does $\lim\limits_{x\to c} f(x)$ exist?

$0\le x<1,\ 1<x<2$

b) At what point(s) c does only the left-hand limit exist?

$x=1,\ x=2$

c) At what point(s) c does only the right-hand limit exist?

$x=0,\ x=1$

16. A water balloon dropped from the roof of a small building falls $y=4.9t^2$ m in t sec. Suppose you wanted to know the speed of the water balloon at exactly $t=2$ seconds. Originally, we used values of t really close to 2 and found the average rate of change between them. Let's try something a little different …

a) Instead of using a numeric value "close" to 2, what would be the average speed of the balloon between $t=2$ and $t=2+h$? (Simplify the expression as much as you can)

$$\frac{4.9(2+h)^2-4.9(2)^2}{2+h-2}=\frac{4.9(4+4h+h^2)-4.9(4)}{h}$$

$(2+h)(2+h)$

$4+4h+h^2$

$$\frac{\cancel{4.9(4)}+4.9(4h)+4.9(h)^2-\cancel{4.9(4)}}{h}\qquad\frac{\cancel{h}(4.9(4)-4.9(h))}{\cancel{h}}$$

$$\boxed{4.9(4)-4.9(h)}$$

b) To find the speed of the balloon at $t=2$, it is tempting to simply plug in $h=0$, however, this yields $\frac{0}{0}$, which is an "indeterminate form". We CAN however, evaluate your simplified expression from part a using limit as $h\to 0$. Evaluate this limit.

$4.9(4)-4.9(h)$

$4.9(4)-4.9(0)$

$4.9(4)$

19.6

~~c) Now find and compare the speed of the balloon at $t=2$ like you did earlier in question #4.~~

AP Calculus
2.1 Worksheet Day 2 1–5

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. When evaluating limits, what does it mean if direct substitution gives you $\frac{7}{0}$? limit grows w/o bound (asymptote)

2. When evaluating limits, what does it mean if direct substitution gives you $\frac{0}{0}$? indeterminate – DO SOMETHING ELSE

3. What are the methods (options) for dealing with the result $\frac{0}{0}$?
rewrite fraction to cancel out 0 in denominator | cancel like factors | rationalize numerator

4. Evaluate the following limits algebraically.

$9-3-6=\frac{0}{0}$ a) $\lim_{x\to 3}\frac{x^2-x-6}{x-3} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}}$

$= \lim_{x\to 3}(x+2)$

$\boxed{=5}$

$\frac{0}{0}$ b) $\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}\left(\frac{2+x}{2+x}\right) = \lim_{x\to 0}\frac{\frac{2}{2(2+x)}-\frac{(2+x)}{2(2+x)}}{x}$

$= \lim_{x\to 0}\frac{\frac{2-2-x}{2(2+x)}}{x} = \lim_{x\to 0}\frac{-x}{2(2+x)}\cdot\frac{1}{\cancel{x}}$

$= \lim_{x\to 0}\frac{-1}{2(2+x)} = \frac{-1}{2(2+0)} = \boxed{\frac{-1}{4}}$

c) $\frac{0}{0}$ $\lim_{x\to 0}\frac{\sqrt{2x+1}-1}{x}\left(\frac{\sqrt{2x+1}+1}{\sqrt{2x+1}+1}\right)$

$\lim_{x\to 0}\frac{(2x+1)-1}{x(\sqrt{2x+1}+1)} = \lim_{x\to 0}\frac{2\cancel{x}}{\cancel{x}(\sqrt{2x+1}+1)}$

$= \lim_{x\to 0}\frac{2}{\sqrt{2x+1}+1} = \frac{2}{\sqrt{0+1}+1} = \frac{2}{1+1} = \boxed{1}$

$\frac{0}{0}$ d) $\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}\left(\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}\right) = \lim_{x\to 4}\frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$= \lim_{x\to 4}\frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x+5}+3)} = \lim_{x\to 4}\frac{1}{\sqrt{x+5}+3}$

$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$

e) $\frac{0}{0}$ $\lim_{x\to 1}\frac{x-1}{x^2-1} = \lim_{x\to 1}\frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

$= \lim_{x\to 1}\frac{1}{(x+1)} = \boxed{\frac{1}{2}}$

f) $\lim_{x\to 0}\frac{(4+x)^2-16}{x} = \lim_{x\to 0}\frac{\cancel{16}+8x+x^2-\cancel{16}}{x}$

$= \lim_{x\to 0}\frac{\cancel{x}(8+x)}{\cancel{x}} = \lim_{x\to 0} 8+x = \boxed{8}$

g) $\lim_{t\to 2}\frac{t^2-3t+2}{t^2-4} = \lim_{t\to 2}\frac{\cancel{(t-2)}(t-1)}{\cancel{(t-2)}(t+2)}$

$= \lim_{t\to 2}\frac{(t-1)}{(t+2)} = \boxed{\frac{1}{4}}$

$(2+x)(2+x)(2+x)$
$(4+4x+x^2)(2+x)$
$8+8x+2x^2+4x+4x^2+x^3$

h) $\lim_{x\to 0}\frac{(2+x)^3-8}{x} = \lim_{x\to 0}\frac{x^3+6x^2+12x+\cancel{8}-\cancel{8}}{x}$

$\lim_{x\to 0}\frac{\cancel{x}(x^2+6x+12)}{\cancel{x}} = \lim_{x\to 0} x^2+6x+12$

$= \boxed{12}$

One of the limits you should know is $\lim_{x\to 0}\frac{\sin x}{x} = \underline{\ 1\ }$. This limit ONLY works when the denominator matches the inside of the sine function. If they do not match, you cannot change the inside of a sine function without a trig identity. Your goal will be to correctly show the algebra in order to use this limit.

5. Evaluate each of the following limits analytically. *Be sure to show ALL steps in your evaluation.*

a) $\lim_{x\to 0}\frac{\sin x}{5x} = \lim_{x\to 0}\underbrace{\frac{\sin x}{x}}\cdot\frac{1}{5} = 1\cdot\frac{1}{5} = \frac{1}{5}$

b) $\lim_{x\to 0}\frac{\sin 5x}{x}\cdot\frac{5}{5} = \lim_{x\to 0}\underbrace{\frac{\sin 5x}{5x}}\cdot\frac{5}{1} = 1\cdot 5 = \boxed{5}$

c) $\lim_{x\to \pi/4}\frac{\sin\left(x-\frac{\pi}{4}\right)}{\left(x-\frac{\pi}{4}\right)} = 1$

match already

d) $\lim_{x\to 0}\frac{3\sin 4x}{\sin 3x} = \lim_{x\to 0}\frac{\sin 4x\cdot 3}{\sin 3x}\cdot\frac{x\cdot 4x}{4x\cdot x}$

$= \lim_{x\to 0}\underbrace{\frac{\sin 4x}{4x}}\cdot\underbrace{\frac{3x}{\sin 3x}}\cdot 4\cancel{x}$

$1\cdot\lim_{x\to 0} = 1\cdot 4 = 4$

6. Evaluate each of the following by combining properties of limits and your algebra skills.

a) $\lim_{x\to 0}\frac{x+\sin x}{x} = \lim_{x\to 0}\frac{x}{x}+\frac{\sin x}{x} = \lim_{x\to 0} 1+1 = \boxed{2}$

b) $\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\sin x}{\cos x}\cdot\frac{1}{x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x} = 1\cdot 1 = \boxed{1}$

c) $\lim_{x\to 0}\frac{\sin x}{2x^2-x} = \lim_{x\to 0}\frac{\sin x}{x(2x-1)} = \lim_{x\to 0}\underbrace{\frac{\sin x}{x}}\cdot\frac{1}{2x-1} = 1\cdot -1 = \boxed{-1}$

d) $\lim_{x\to 0}\frac{\sin^2 x}{x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{x}\cdot\frac{x}{x} = \lim_{x\to 0}\underbrace{\frac{\sin x}{x}\cdot\frac{\sin x}{x}}\cdot\frac{x}{1} = 1\quad 1\cdot 0 = \boxed{0}$

7. Consider $\lim_{x\to 0}\frac{f(x)-f(0)}{x^2} = \frac{0}{0}$

a) If you use direct substitution, what result do you get? $\frac{0}{0}$

b) Evaluate the limit if $f(x) = 2x^2+1$.

$\lim_{x\to 0}\frac{2x^2+1-1}{x^2} = \lim_{x\to 0}\frac{2x^2}{x^2} = \boxed{2}$

8. If $a\neq 0$, then $\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4} =$ $\lim_{x\to a}\frac{\cancel{(x^2-a^2)}}{(x^2-a^2)(x^2+a^2)} = \lim_{x\to a}\frac{1}{x^2+a^2} = \frac{1}{a^2+a^2} = \frac{1}{2a^2}$

9. Evaluate the following limits analytically (all mixed up):

$\frac{0}{0}$ a) $\lim_{x\to 0}\frac{\frac{3}{4+x}-\frac{3}{4}}{x} = \lim_{x\to 0}\frac{\frac{12}{4(4+x)}-\frac{3(4+x)}{4(4+x)}}{x}$

$= \lim_{x\to 0}\frac{\frac{12-12-3x}{4(4+x)}}{x} = \lim_{x\to 0}\frac{-3\cancel{x}}{4(4+x)}\cdot\frac{1}{\cancel{x}} = \lim_{x\to 0}\frac{-3}{4(4+x)}$

$= \frac{-3}{4\cdot 4} = \boxed{-\frac{3}{16}}$

$\frac{0}{0}$ b) $\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2} = \lim_{x\to 0}\frac{x^2(5x+8)}{x^2(3x^2-16)}$

$= \lim_{x\to 0}\frac{5x+8}{3x^2-16} = \frac{5(0)+8}{3(0)^2-16} = \frac{8}{-16} = \boxed{-\frac{1}{2}}$

c) $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4}$ DIRECT SUBST. YAY!

$\frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \frac{2}{-1} = \boxed{-2}$

$\frac{0}{0}$ d) $\lim_{x\to 0}\frac{x^2-3x}{x}$ $\lim_{x\to 0}\frac{x(x-3)}{x} = \lim_{x\to 0} x-3 = 0-3 = \boxed{-3}$

$\frac{1}{0}$ e) $\lim_{x\to 1}\frac{x}{x^2-x}$ $\lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(x-1)} = \lim_{x\to 0}\frac{1}{x-1}$

$= \frac{1}{0}$

$=$ DNE (asymptote

DNE
asymptote

f) $\lim_{x\to 0}\frac{\sin 2x}{x}\cdot\frac{2}{2} = \underbrace{\lim_{x\to 0}\frac{\sin 2x}{2x}}_{1}\cdot\frac{2}{1}$

$1\cdot\frac{2}{1}$

$= \boxed{2}$

g) $\lim_{x\to 0}\frac{\sin 7x}{3x}\cdot\frac{7}{7} = \underbrace{\lim_{x\to 0}\frac{\sin 7x}{7x}}_{1}\cdot\frac{7}{3}$

$1\cdot\frac{7}{3} = \frac{7}{3}$

$\frac{0}{0}$ h) $\lim_{x\to 4}\frac{x^2-5x+4}{x^2-2x-8} = \lim_{x\to 4}\frac{\cancel{(x-4)}(x-1)}{\cancel{(x-4)}(x+2)}$

$= \lim_{x\to 4}\frac{(x-1)}{(x+2)} = \frac{4-1}{4+2} = \frac{3}{6} = \boxed{\frac{1}{2}}$

12. Evaluate $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$.

♬: *h* is going to 0 … not *x* … so treat this as if *h* is the variable … your final answer will have a *x* in it.

$\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x+0 = \boxed{2x}$

13. Suppose $g(x)=\begin{cases}2-x, & \text{if } x\le 1\\ \frac{x}{2}+1, & \text{if } x>1\end{cases}$.

a) $\lim_{x\to 1^-} g(x) =$

left
plug 1 in
top fctn

$2-1 = \boxed{1}$

b) $\lim_{x\to 1^+} g(x) =$

right
plug 1 in
top fctn

$\frac{1}{2}+1 = \boxed{\frac{3}{2}}$

c) $\lim_{x\to 1} g(x) =$

$\boxed{\text{DNE}}$
because
g(x) approaches
2 different
values
as $x\to 1$

d) $g(1) =$

plug in
top
fctn

$\boxed{1}$

AP Calculus
2.2 Worksheet

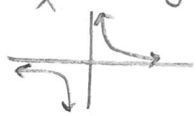
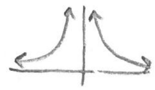
1–8, skip 5, 6   1–12 skip 5, 6

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Answer the following questions:

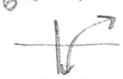
   a) How do you find horizontal asymptotes? look at end behavior $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$
   horizontal asymptotes occur when $y \to \#$ as $x \to \pm\infty$

   b) Which of the parent functions have horizontal asymptotes? List the function(s) and asymptote(s)
   $y = b^x$ H.A. $y=0$ $y=\frac{1}{x}$ H.A. $y=0$ $y=\frac{1}{x^2}$ H.A. $y=0$

   c) How do you find vertical asymptotes?
   find x-values that make numer $= \#$ and denom $= 0$
   find values $x=a$ that make behavior of $f(x)$ true for $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$

   d) Which of the parent functions have vertical asymptotes? List the function(s) and asymptote(s)
   $y=\log_b x$ V.A. $x=0$ $y=\frac{1}{x}$ V.A. $x=0$ $y=\frac{1}{x^2}$ V.A. $x=0$ $y=\tan x$



   e) When must you look for oblique (slanted) asymptotes? How do you find them?
   When degree numerator > degree denom.
   To find them, do polynomial long division

2. For each of the following, find (i) $\lim_{x\to\infty} f(x)$ and (ii) $\lim_{x\to-\infty} f(x)$. Then (iii) identify all horizontal asymptotes, if any.

   a) $f(x)=\dfrac{x-2}{2x^2+3x-5}$
   (i) $\lim_{x\to\infty} f(x) = 0$
   (ii) $\lim_{x\to-\infty} f(x) = 0$
   (iii) $y=0$ H.A.

   b) $f(x)=\dfrac{4x^3-2x+1}{x^2-2x+1}$
   (i) $\lim_{x\to\infty} f(x) = \infty$
   (ii) $\lim_{x\to-\infty} f(x) = -\infty$
   (iii) N/A

   c) $f(x)=\dfrac{3x^2-x+5}{x^2-4}$
   (i) $\lim_{x\to\infty} f(x) = 3$
   (ii) $\lim_{x\to\infty} f(x) = 3$
   (iii) $y=3$ H.A.

   d) $f(x)=\dfrac{e^{-x}}{x}$
   $\lim_{x\to\infty} f(x) = 0$
   $\lim_{x\to-\infty} f(x) = -\infty$
   $y=0$ HA

   e) $f(x)=\dfrac{|x|}{x}$
   $\lim_{x\to\infty} f(x) = 1$
   $\lim_{x\to-\infty} f(x) = -1$
   $y=1$ HA
   $y=-1$

   f) $f(x)=\dfrac{\sin x}{2x^2+x}$
   $\lim_{x\to\infty} f(x) = 0$
   $\lim_{x\to-\infty} f(x) = 0$
   $y=0$ H.A.

3. One of the functions in 2a – 2c has a slanted (oblique) asymptote. Explain why, and then find the asymptote.
   polynomial long division

$$\begin{array}{r} \boxed{4x+8} \\ x^2-2x+1 \,\big)\, 4x^3+0x^2-2x+1 \\ -(4x^3-8x^2+4x) \\ \hline 8x^2-6x+1 \\ -(8x^2+16x) \end{array}$$

   Slant asymptote $y=4x+8$

4. For each of the following, (i) find the vertical asymptotes of the graph of $f(x)$ and (ii) describe the behavior of the graph of $f(x)$ to the left and right of each asymptote.

a) $f(x)=\dfrac{1}{x-3}$

V.A. $X=3$

$\lim\limits_{x\to 3^-} f(x) = -\infty$ $\quad$ $\lim\limits_{x\to 3^+} f(x) = \infty$

b) $f(x)=\dfrac{1}{x^2-4}$

VA = $X=2$, $X=-2$

$\lim\limits_{x\to -2^-} f(x) = \infty$ $\quad$ $\lim\limits_{x\to 2^-} f(x) = -\infty$

$\lim\limits_{x\to -2^+} f(x) = -\infty$ $\quad$ $\lim\limits_{x\to 2^+} f(x) = \infty$

c) $f(x)=\dfrac{1-x}{2x^2-5x-3}$

VA $X=-1/2$ $X=3$

$(2x+1)(x-3)$

$\lim\limits_{x\to -1/2^-} f(x) = \infty$ $\quad$ $\lim\limits_{x\to 3^-} f(x) = -\infty$

$\lim\limits_{x\to -\frac{1}{2}^+} f(x) = -\infty$ $\quad$ $\lim\limits_{x\to 3^+} f(x) = \infty$

5. Find the limit of $g(x)$ as (i) $x\to\infty$, (ii) $x\to-\infty$, (iii) $x\to 0^-$, and (iv) $x\to 0^+$

a) $g(x)=\begin{cases}\frac{1}{x} & \text{if } x<0\\ \frac{2x-3}{x+1} & \text{if } x\geq 0\end{cases}$

$\lim\limits_{x\to\infty} g(x) = \infty$ $\quad$ $\lim\limits_{x\to 0^-} g(x) = -\infty$

$\lim\limits_{x\to -\infty} g(x) = 0$ $\quad$ $\lim\limits_{x\to 0^+} g(x) = -3$

b) $g(x)=\begin{cases}\frac{3x}{x+1} & \text{if } x\leq 0\\ \frac{1}{x^2} & \text{if } x>0\end{cases}$

$\lim\limits_{x\to\infty} g(x) =$ $\quad$ $\lim\limits_{x\to 0^-} g(x) =$

$\lim\limits_{x\to -\infty} g(x) =$ $\quad$ $\lim\limits_{x\to 0^+} g(x) =$

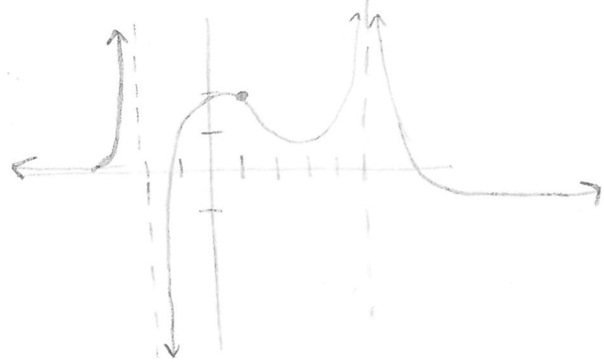
6. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim\limits_{x\to 1} f(x)=2$ $\quad$ $\lim\limits_{x\to 5^-} f(x)=\infty$

$\lim\limits_{x\to 5^+} f(x)=\infty$ $\quad$ $\lim\limits_{x\to\infty} f(x)=-1$

$\lim\limits_{x\to -\infty} f(x)=0$ $\quad$ $\lim\limits_{x\to -2^-} f(x)=\infty$

$\lim\limits_{x\to -2^+} f(x)=-\infty$

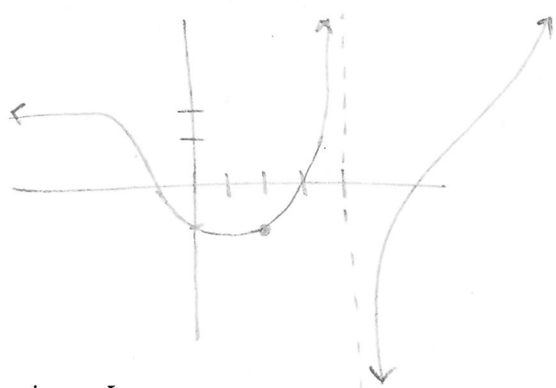


7. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim\limits_{x\to 2} f(x)=-1$ $\quad$ $\lim\limits_{x\to 4^+} f(x)=-\infty$

$\lim\limits_{x\to 4^-} f(x)=\infty$ $\quad$ $\lim\limits_{x\to\infty} f(x)=\infty$

$\lim\limits_{x\to -\infty} f(x)=2$



8. Explain why there is no value L for which $\lim\limits_{x\to\infty}\sin x = L$.

Sin will never approach a value, it will keep oscillating between 1, & -1.

9. Let $f(x)=\dfrac{\cos x}{x}$.

a) Find the domain and range of *f*. domain $(-\infty,0)\cup(0,\infty)$
range $(-\infty,\infty)$

b) Is *f* even, odd, or neither? Justify your response. odd 180° rotation symmetry

c) Find $\lim_{x\to\infty} f(x)$. Give a reason for your answer.

$\lim_{x\to\infty} f(x)=0$ it oscillates around 0, but each time gets

10. If *k* is a positive integer, then $\lim_{x\to\infty}\dfrac{x^k}{e^x}=$? Explain your answer.

[Try letting $k=2$ … what about $k=10$? … what about $k=1000$?]

$\lim_{x\to\infty}\dfrac{x^k}{e^x}=0$ as $x\to\infty$, e^x will be larger than x^k, so the fraction will go to 0.

11. ***Investigate*** using tables and graphs to determine the value of each limit: $\lim_{x\to\infty}\dfrac{3x-2}{\sqrt{2x^2+1}}$ and $\lim_{x\to-\infty}\dfrac{3x-2}{\sqrt{2x^2+1}}$

$\lim_{x\to\infty}\dfrac{3x-2}{\sqrt{2x^2+1}}=2.12$ $\lim_{x\to-\infty}\dfrac{3x-2}{\sqrt{2x^2+1}}=-2.12$

$\dfrac{3x}{\sqrt{2x^2}}=\dfrac{3x}{\sqrt{2}x}=\dfrac{3}{\sqrt{2}}=2.12$

12. Evaluate each of the following limits using all methods learned from this chapter.

a) $\lim_{x\to\infty}\left(\dfrac{2}{x}+1\right)\left(\dfrac{5x^2-1}{x^2}\right)$

$(0+1)(5)=5$

b) $\lim_{n\to\infty}\dfrac{3n^3-5n}{n^3-2n^2+1}=$ 3

c) $\lim_{x\to\infty}\left(5-\dfrac{2}{x^2}\right)=$

$5-0=5$

d) $\lim_{x\to\pi/2}\sec x$ $\dfrac{1}{\cos x}$ $\dfrac{1}{0}$ = DNE

limit approaches 2 diff. values of y on each side of $x=\pi/2$

e) $\lim_{x\to\infty} e^{-x}\cos x$

$(0)(\text{DNE})=0$

f) $\lim_{x\to 7/2^+}\operatorname{int}(2x-1)$

3.5^+ $(2(3.5)-1)=6$

g) $\lim_{x\to\infty} \dfrac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}}$ $\dfrac{1}{1} = \boxed{1}$

h) $\lim_{x\to\infty} \dfrac{4n^3}{n^2+10000n} = \boxed{\infty}$

i) $\lim_{x\to 0} \dfrac{\sin 2x}{4x} = \lim_{x\to 0} \underbrace{\dfrac{\sin 2x}{2x}}_{1} \cdot \dfrac{1}{2}$

$1 \cdot \dfrac{1}{2} = \boxed{\dfrac{1}{2}}$

j) $\lim_{x\to 0} \dfrac{\frac{2}{2}\frac{1}{2+x} - \frac{1}{2}\frac{2+x}{2+x}}{x} = \lim_{x\to 0} \dfrac{\frac{2}{2(2+x)} - \frac{(2+x)}{2(2+x)}}{x}$

$= \lim_{x\to 0} \dfrac{\cancel{2}\cancel{-2}-x}{2(2+x)} \cdot \dfrac{1}{x} = \lim_{x\to 0} \dfrac{-\cancel{x}}{2(2+x)} \cdot \dfrac{1}{\cancel{x}}$

$= \lim_{x\to 0} \dfrac{-1}{2(2+x)} = \dfrac{-1}{2(2)} = \boxed{-\dfrac{1}{4}}$

k) $\lim_{x\to\infty} \dfrac{x\sin x + 2\sin x}{2x^2}$

$\dfrac{(x+2)(\sin x)}{2x^2} = \boxed{0}$

l) $\lim_{x\to -2} \dfrac{x^2+1}{3x^2-2x+5}$

direct substitution

$\dfrac{(-2)^2+1}{3(-2)^2-2(-2)+5} = \dfrac{5}{12+4+5} = \dfrac{5}{21}$

AP Calculus
2.3 Worksheet

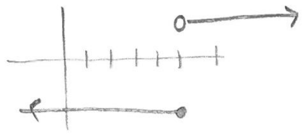
All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the definition of continuity?

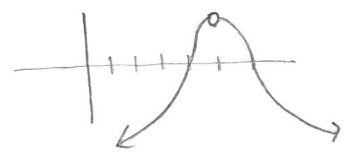
   $\lim_{x \to c} f(x)$ exists $f(c)$ exists $\lim_{x \to c} f(x) = f(c)$

2. Sketch a possible graph for each function described.

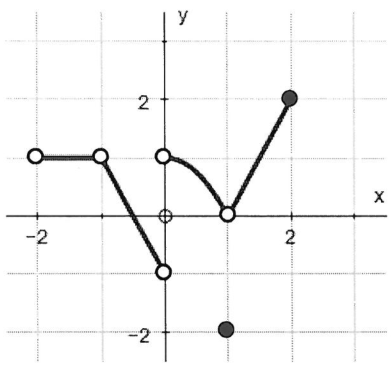
   a) $f(5)$ exists, but $\lim_{x \to 5} f(x)$ does not exist.

   b) The $\lim_{x \to 5} f(x)$ exists, but $f(5)$ does not exist.

3. Use the function $g(x)$ defined and graphed below to answer the following questions.



$$g(x) = \begin{cases} 1 & \text{if } -2 < x < -1 \\ -2x - 1 & \text{if } -1 < x < 0 \\ 1 - x^2 & \text{if } 0 < x < 1 \\ -2 & \text{if } x = 1 \\ 2x - 2 & \text{if } 1 < x \le 2 \end{cases}$$

a) Does $g(1)$ exist?

yes $g(1) = -2$

b) Does $\lim_{x \to 1} g(x)$ exist?

yes $\lim_{x \to 1} g(x) = 0$

c) Does $\lim_{x \to 1} g(x) = g(1)$?

NO

d) Is g continuous at $x = 1$?

No

e) Is g defined at $x = -1$?

no

$g(-1)$ does not exist

f) Is g continuous at $x = -1$?

NO, $g(x)$ not defined @ $x = -1$

g) For what values of x is g continuous?

$(-2, -1) \cup (-1, 0) \cup (0, 1) \cup (1, 2]$

h) What value should be assigned to $g(-1)$ to make the extended function continuous at $x = -1$?

$g(-1) = 1$

i) What new value should be assigned to $g(1)$ to make the new function continuous at $x = 1$?

$g(1) = 0$ (cross out -2)

j) Is it possible to extend g to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

NO, $\lim_{x \to 0^-} g(x) = -1$ $\lim_{x \to 0^+} g(x) = 1$

4. Let $f(x)=\begin{cases} x^2-1 & ;x<3 \\ 2ax & ;x\geq 3 \end{cases}$. Find a value of a so that the function f is continuous.

Using the definition of continuity, justify your response.

$3^2-1=2ax$ at $x=3$

$9-1=2a(3)$

$9-1=6a$

$8=6a$

$a=\frac{8}{6}$

$\boxed{a=\frac{4}{3}}$

$\lim_{x\to 3^-} f(x)=8$ ✓

$\lim_{x\to 3^+} f(x)=\frac{8}{3}(3)=8$ ✓

$f(3)=\frac{8}{3}(3)=8$ ✓

5. Let $f(x)=\begin{cases} 2x+3 & ;x\leq 2 \\ kx+1 & ;x>2 \end{cases}$. Find a value of k so that the function f is continuous.

Using the definition of continuity, justify your response.

let $x=2$ $\quad 2(2)+3=K(2)+1$

$4+3=2K+1$

$7=2K+1$

$6=2K$

$3=K$

$\lim_{x\to 2^-} f(x)=7$ ✓

$\lim_{x\to 2^+} f(x)=3(2)+1=7$ ✓

$f(2)=2(2)+3=7$ ✓

6. Let $f(x)=\begin{cases} x^2-a^2x & ;x<2 \\ 4-2x^2 & ;x\geq 2 \end{cases}$. Find all values of a that make f continuous at 2.

Using the definition of continuity, justify your response.

$4-2(2)^2=4-2(4)=4-8=-4$

$-4=x^2-a^2x \quad (x=2)$

$-4=2^2-a^2(2)$

$-4=4-2a^2$

$-8=-2a^2$

$4=a^2$

$\boxed{a=\pm 2}$

$\lim_{x\to 2^-} f(x)=-4$ ✓

$\lim_{x\to 2^+} f(x)=-4$ ✓

$f(2)=-4$ ✓

7. If $f(x)=\begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & \text{if } x\neq 2 \\ k+3 & \text{if } x=2 \end{cases}$, and if f is continuous at $x=2$, then $k=$?

Using the definition of continuity, justify your response.

$2x+5-(x+7)$

$$\lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\cdot\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}} = \lim_{x\to 2}\frac{2x+5-x-7}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})} = \lim_{x\to 2}\frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})}$$

$$=\frac{1}{\sqrt{9}+\sqrt{9}}=\boxed{\frac{1}{3+3}=\frac{1}{6}}$$

$K+3=\frac{1}{6}$

$K=\frac{1}{6}-3\left(\frac{18}{6}\right)=\boxed{-\frac{17}{6}}$

8. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-4}{x+2}$, when $x\neq -2$, then $f(-2)=$

Use the definition of continuity to justify your response.

$$\lim_{x\to -2}\frac{x^2-4}{x+2}=\lim_{x\to -2}\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}}=\lim_{x\to -2}(x-2)=-2-2=-4$$

$f(-2)=-4$

9. Let f be the function defined by the following:

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

possible pts of discontinuity $x=0, x=1, x=2$

For what values of x is f NOT continuous? Use the definition of continuity to explain why.

If $f(x)$ is cont. @ $x=0$, then

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\lim_{x\to 0^-} \sin x = \lim_{x\to 0} x^2 = f(0)$

$0 = 0^2 = 0^2$

$0 = 0 = 0$

$\therefore f(x)$ is CONT. @ $x=0$

If $f(x)$ is cont. @ $x=1$, then

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\lim_{x\to 1^-} x^2 = \lim_{x\to 1^+} 2-x = f(1)$

$1^2 = 2-1 = 2-1$

$1 = 1 = 1$

$\therefore f(x)$ is continuous @ $x=1$

If $f(x)$ is cont @ $x=2$, then

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\lim_{x\to 2^-} 2-x = \lim_{x\to 2^+} x-3 = f(2)$

$2-2 = 2-3 = 2-3$

$0 \ne -1 \ne -1$

$\therefore f(x)$ is NOT CONT @ $x=2$

10. Determine the points of discontinuity and identify their type for each of the following functions:

a) $y=\dfrac{1}{(x+2)^2}$

$x=-2$ is a pt of discontinuity

nonremovable (infinite)

b) $y=\dfrac{x-1}{x^2-4x+3}$

$(x-1)(x-3)$

not continuous @ $x=1, x=3$

$x=1$ removable disc. (hole)

$x=3$ nonremov (infinite)

c) $y=\dfrac{|x|}{x}$

not continuous @ $x=0$

nonremovable (jump)

11. Write an extended function so that the given function is continuous at the indicated point.

a) $h(x)=\dfrac{\sin(5x)}{x}$ at $x=0$

$\lim_{x\to 0} \dfrac{\sin 5x}{x}\cdot\dfrac{5}{5} = \underbrace{\lim_{x\to 0}\dfrac{\sin 5x}{5x}}_{1}\cdot 5$

$1\cdot 5 = 5$

$$\boxed{h(x)=\begin{cases}\dfrac{\sin(5x)}{x}, & x\ne 0\\ 5, & x=0\end{cases}}$$

b) $k(x)=\dfrac{x-4}{\sqrt{x}-2}$ at $x=4$

$\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4}\dfrac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}}$

$=\lim_{x\to 4}\sqrt{x}+2 = 4$

$$K(x)=\begin{cases}\dfrac{x-4}{\sqrt{x}-2}, & x\ne 4\\ 4, & x=4\end{cases}$$

12. Let f be the function given by $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers?

Because this is MC, only possible values of a are $a=1,2,4$

A) None B) 1 only C) 2 only D) 4 only E) 1 and 4

if $a=1$ then $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-1}=\dfrac{(x-1)(x^2-4)}{(x-1)(x+1)}$, $f(x)$ has discontinuity @ $x=-1$ (nonremov) $x=1$ (remov)

if $a=2$ then $f(x)=\dfrac{(x-1)(x^2-4)}{(x^2-2)}$ $f(x)$ has discontinuity @ $x=\pm\sqrt{2}$ (nonremov)

if $a=4$, then $f(x)=\dfrac{(x-1)(x^2-4)}{(x^2-4)}$ $f(x)$ has discontinuity @ $x=\pm 2$ (both removable)

13. Let $g(x)=\frac{x^2+5x+6}{x^2+7x+10}$. $g(x)=\frac{(x+2)(x+3)}{(x+2)(x+5)}$

a) Find the domain of $g(x)$.

$x \neq -2 \quad x \neq -5$

$(-\infty,-5)\cup(-5,-2)\cup(-2,\infty)$

b) Find the $\lim\limits_{x\to c} g(x)$ for all values of c where $g(x)$ is not defined.

$\lim\limits_{x\to -5} g(x)=\frac{(-5+2)(-5+3)}{(-5+2)(-5+5)}=\frac{6}{0}=\text{DNE}$

as $x \to 5$, $g(x)$ grows w/o bound

$\lim\limits_{x\to -2} g(x)=\frac{(x+2)(x+3)}{(x+2)(x+5)}=\frac{-2+3}{-2+5}$ $\boxed{\frac{1}{3}}$

c) Find any horizontal asymptotes and justify your response.

end behavior $\lim\limits_{x\to\pm\infty} \frac{x^2+5x+6}{x^2+7x+10}=1$ H.A. $y=1$

d) Find any vertical asymptotes and justify your response.

$y=-5$ is a V.A. ($g(x)$ grows w/o bound there)

($y=-2$ is a hole)

e) Write an extension to the function so that $g(x)$ is continuous at $x = -2$.
<u>Use the definition of continuity</u> to justify your response.

If $g(x)$ is continuous @$x=-2$, $\lim\limits_{x\to -2} g(x)=g(-2)$

$$g(x)=\begin{cases}\frac{x^2+5x+6}{x^2+7x+10}, & x\neq -2\\ \frac{1}{3}, & x=2\end{cases}$$

14. Without using a picture, give a <u>written</u> explanation of why the function $f(x)=x^2-4x+3$ has a zero in the interval [2, 4].

$f(2)=2^2-4(2)+3=-1$

$f(4)=4^2-4(4)+3=3$

Since $f(x)$ is continuous on $[2,4]$, it must take on every y-value between -1 and 3 (the IVT tells us this). Since 0 is between -1 & 3, there must exist c such that $f(c)=0$

15. Without using a picture, give a <u>written</u> explanation of why the function $f(x)=x^2+2x-3$ must equal 3 at least once in the interval [0, 2].

$f(0)=-3$

$f(2)=5$

Since $f(x)$ is continuous on $[0,2]$, the IVT holds. The IVT says each y-value on interval $[-3,5]$ is represented. Since 3 is between -3 & 5, there is some c in $[0,2]$ where $f(c)=3$

Skip 16. Let $h(x)=\begin{cases}3x^2-4, & \text{if } x\le 2\\ 5+4x, & \text{if } x>2\end{cases}$.

a) What is $h(0)$? $h(0)=3(0)^2-4$ $=-4$

b) What is $h(4)$? $h(4)=5+4(4)$ $=21$

c) On the interval [0, 4], there is no value of x such that $h(x) = 10$ even though $h(0) < 10$ and $h(4) > 10$. Explain why this result does not contradict the IVT.

fctn is not continuous on $[0,4]$

AP Calculus
2.4 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is a difference quotient? $\frac{f(a+h)-f(a)}{h}$

2. How do you find the slope of a curve (aka slope of the tangent line to a curve) when $x = a$?

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

3. What is a *normal line* ?

a normal line is $\perp$ to a tangent line.

4. What is the difference between the AVERAGE RATE OF CHANGE and INSTANTANEOUS RATE OF CHANGE?

avg R.O.C = slope of secant line 2pts $m = \frac{y_2 - y_1}{x_2 - x_1}$ | instantaneous R.O.C = slope of curve at one point $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

5. Find the average rate of change of each function over the indicated interval. → Slope!

a) $h(x) = e^x$ on $[-2, 0]$

avg R.O.C. $= \frac{h(0)-h(-2)}{0--2} = \frac{e^0 - e^{-2}}{2} = \boxed{\frac{1-e^{-2}}{2}}$

b) $k(x) = 2 + \sin x$ on $[-\pi/2, \pi/2]$

avg R.O.C $= \frac{k(\pi/2)-k(-\pi/2)}{\pi/2 - -\pi/2} = \frac{2+\sin\pi/2 - (2+\sin(-\pi/2))}{\pi} = \frac{2+1-(2-1)}{\pi} = \boxed{\frac{2}{\pi}}$

c) $f(x) = x^2 - x$ on $[1, 3]$

avg R.O.C $\frac{f(3)-f(1)}{3-1} = \frac{3^2-3-(1^2-1)}{2} = \frac{6-0}{2} = \boxed{3}$

6. Let $f(x) = x^3$.

a) Write and simplify an expression for $f(a+h)$.

$f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$

b) Find the slope of the curve at $x = a$.

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{a^3+3a^2h+3ah^2+h^3-a^3}{h} = \lim_{h\to 0} \frac{3a^2h+3ah^2+h^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3a^2+3ah+h^2)}{\cancel{h}} = \lim_{h\to 0} 3a^2 + 3a\overset{\to 0}{\cancel{h}} + \overset{\to 0}{\cancel{h^2}} = \underline{3a^2}$$

formula for slope at any point a!

c) When does the slope equal 12?

$3a^2 = 12$
$a^2 = 4$
$a = \pm 2$ so when $x = \pm 2$, slope will be 12.

d) Write the equation of the tangent line to the curve at $x = 4$ $\underline{y - 64 = 48(x-4)}$

$x = 4$ $y = 64$ $m = 48$

e) Write the equation of the normal line to the curve at $x = 4$ $\underline{y - 64 = -\frac{1}{48}(x-4)}$

7. Let $g(x)=\sqrt{x}$

a) Find the average rate of change from $x = 4$ to $x = 9$. avg R.O.C. $\frac{g(9)-g(4)}{9-4} = \frac{\sqrt{9}-\sqrt{4}}{5} = \frac{3-2}{5} = \frac{1}{5}$

Slope between 2pts.

b) Find the instantaneous rate of change at $x = 9$.

instant. R.O.C = limit of diff quot.

$$\lim_{h\to0}\frac{g(9+h)-g(9)}{h} = \lim_{h\to0}\frac{\sqrt{9+h}-\sqrt{9}}{h} = \lim_{h\to0}\frac{\sqrt{9+h}-3}{h}\cdot\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} = \lim_{h\to0}\frac{(9+h)-9}{h(\sqrt{9+h}+3)}$$

$$= \lim_{h\to0}\frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)} = \lim_{h\to0}\frac{1}{\sqrt{9+h}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

c) Write the equation of the tangent line when $x = 9$

$x=9$
$y=3$
$m=\frac{1}{6}$

$$y-3=\tfrac{1}{6}(x-9)$$

d) Write the equation of the normal line when $x = 9$.

$\perp$ to tangent

$x=9$
$y=3$
$m=-6$

$$y-3=-6(x-9)$$

8. Let $y=\frac{1}{x-1}$. Find the slope of the curve at $x = 2$. Using this slope, write the equation of the tangent line and the equation of the normal line at that point.

instantaneous R.O.C.

κ need common denom

$$\lim_{h\to0}\frac{y(2+h)-y(2)}{h} = \lim_{h\to0}\frac{\frac{1}{2+h-1}-\frac{1}{2-1}}{h} = \lim_{h\to0}\frac{\frac{1}{1+h}-1\cdot\frac{(1+h)}{(1+h)}}{h} = \lim_{h\to0}\frac{\frac{1}{1+h}-\frac{1+h}{1+h}}{h}$$

$$= \lim_{h\to0}\frac{1-1-h}{1+h}\cdot\frac{1}{h} = \lim_{h\to0}\frac{-\cancel{h}}{1+h}\cdot\frac{1}{\cancel{h}} = \lim_{h\to0}\frac{-1}{1+h} = \frac{-1}{1} = -1$$

Slope of curve @ $x=2$

9. Let $y=x^2-3x-2$. Find the slope of the curve at $x = 0$. Using this slope, write the equation of the tangent line and the equation of the normal line at that point.

$$\lim_{h\to0}\frac{(0+h)^2-3(0+h)-2-(0^2-3(0)-2)}{h} = \lim_{h\to0}\frac{h^2-3h-\cancel{2}+\cancel{2}}{h} = \lim_{h\to0}\frac{\cancel{h}(h-3)}{\cancel{h}} = \lim_{h\to0}h-3$$

$= -3$

$x=0$
$y=-2$
$m=-3$

tangent line

$$y+2=-3(x-0)$$

$x=0$
$y=-2$
$m=+\frac{1}{3}$

normal line

$$y+2=\tfrac{1}{3}(x-0)$$

10. Find an equation of the tangent line to the graph of $f(x) = \frac{3}{x}$ at $x = 1$.

$x = 1$

$y = 3/1 = 3$

$m = -3$

instant. R.O.C.

$y - 3 = -3(x - 1)$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{1+h} - 3\frac{(1+h)}{(1+h)}}{h} = \lim_{h \to 0} \frac{\frac{3}{1+h} - \frac{3(1+h)}{1+h}}{h} = \lim_{h \to 0} \frac{\frac{3-3-3h}{1+h}}{h}$$

*need common denom

$$\lim_{h \to 0} \frac{-3\cancel{h}}{1+h} \cdot \frac{1}{\cancel{h}} = \lim_{h \to 0} \frac{-3}{1+h^{\to 0}} = -3$$

11. An object is dropped from the top of a 150-m tower. It's height above the ground after t seconds is $150 - 4.9t^2$ m. How fast is the object falling 2 seconds after it is dropped?

what is the instantaneous R.O.C @ $t = 2$

$S(t) = 150 - 4.9t^2$

$$\lim_{h \to 0} \frac{s(a+h) - s(a)}{h} = \lim_{h \to 0} \frac{s(2+h) - s(2)}{h} = \lim_{h \to 0} \frac{150 - 4.9(2+h)^2 - (150 - 4.9(2)^2)}{h}$$

$$= \lim_{h \to 0} \frac{150 - 4.9(4 + 4h + h^2) - (150 - 19.6)}{h} = \lim_{h \to 0} \frac{\cancel{150} - \cancel{19.6} - 19.6h - 4.9h^2 - \cancel{150} + \cancel{19.6}}{h} = \lim_{h \to 0} \frac{\cancel{h}(-19.6 - 4.9h)}{\cancel{h}}$$

$$= \lim_{h \to 0} -19.6 - 4.9h^{\to 0}$$

$$\boxed{= -19.6 \text{ m/sec}}$$

12. What is the rate of change of the area of a circle with respect to the radius when the radius is 4 in?

instant. R.O.C. $r = 4$

$A = \pi r^2$

$$\lim_{h \to 0} \frac{A(4+h) - A(4)}{h} = \lim_{h \to 0} \frac{\pi(4+h)^2 - \pi(4)^2}{h} = \lim_{h \to 0} \frac{\pi(16 + 8h + h^2) - 16\pi}{h} = \lim_{h \to 0} \frac{\cancel{16\pi} + 8h\pi + \pi h^2 - \cancel{16\pi}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(8\pi + \pi h)}{\cancel{h}} = 8\pi + \pi h^{\to 0}$$

$$\boxed{= 8\pi \text{ in}^2/\text{in}}$$

13. At what point is the tangent line to $g(x) = x^2 - 6x + 1$ horizontal?

$\boxed{\text{when is instantaneous slope} = 0?}$

$$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 1 - (x^2 - 6x + 1)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h + \cancel{1} - \cancel{x^2} + \cancel{6x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 6)}{\cancel{h}} = 2x + h^{\to 0} - 6$$

$= 2x - 6$ = slope of tangent line

horizontal: $0 = 2x - 6$

$6 = 2x$

$3 = x$

tangent line is horizontal @ $x = 3$

what is y-coordinate at that pt?

$g(3) = 3^2 - 6(3) + 1$

$= 9 - 18 + 1$

$= -8$

$\therefore$ The tangent line is horiz. @ $(3, -8)$