KEY

AP Calculus
3.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the original limit definition of a derivative?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

2. What is the alternative definition of a derivative?

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

3. Use the original definition of the derivative to find the derivative of each function at the indicated point.

a) $f(x) = \frac{1}{x}$ at $a = 2$

$$f'(x) = \lim_{h\to 0} \frac{\frac{(x)}{(x)}\frac{1}{x+h} - \frac{1}{x}\frac{(x+h)}{(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x}-\cancel{x}-\cancel{h}}{(x)(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+\cancel{h}^{0})} = \frac{-1}{x^2}$$

so $f'(x) = \frac{-1}{x^2}$ at any pt. $x=a$. $\boxed{f'(2) = \frac{-1}{4}}$

b) $K(x) = 3 - x^2$ at $a = -2$.

$$f'(x) = \lim_{h\to 0} \frac{3-(x+h)^2 - (3-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{3-(x^2+2xh+h^2) - 3 + x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3}-\cancel{x^2}-2xh-h^2-\cancel{3}+\cancel{x^2}}{h} = \lim_{h\to 0} \frac{-2xh-h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = -2x-\cancel{h}^{0} = -2x = K'(x)$$

$\boxed{K'(-2) = 4}$

★ Remember you can also apply the point $x=a$ from the beginning.

c) $g(x) = \sqrt{x+1}$ at $a = 3$

$$g'(x) = \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{(\sqrt{x+h+1} + \sqrt{x+1})}{(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h\to 0} \frac{x+h+1-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})} = \lim_{h\to 0} \frac{\cancel{x}+h+\cancel{1}-\cancel{x}-\cancel{1}}{h(\sqrt{x+h+1}+\sqrt{x+1})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+1}+\sqrt{x+1})} = \frac{1}{\sqrt{x+1}+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}} = g'(x)$$

$\boxed{g'(3) = \frac{1}{2\sqrt{4}} = \frac{1}{4}}$

d) $f(x) = x - x^3$ at $a = -1$

$$f'(x) = \lim_{h\to 0} \frac{(-1+h)-(-1+h)^3 - \cancel{(-1-(-1)^3)}}{h}$$

$$= \lim_{h\to 0} \frac{-1+h-(-1+3h-3h^2+h^3)-\cancel{(-1+1)}^{0}}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{1}+h+\cancel{1}-3h+3h^2-h^3}{h} = \lim_{h\to 0} \frac{\cancel{h}(1-3+3h-h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0} -2+3\cancel{h}^{0}-\cancel{h^2}^{0} = \boxed{-2}$$

4. Repeat question 3a – 3c using the alternative definition of the derivative.

$$f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{\frac{(2)}{(2)}\frac{1}{x} - \frac{1}{2}\left(\frac{x}{x}\right)}{x-2}$$

$$= \lim_{x\to 2} \frac{\frac{2}{2x} - \frac{x}{2x}}{x-2} = \lim_{x\to 2} \frac{2-x}{2x} \cdot \frac{1}{x-2} = \lim_{x\to 2} \frac{-1\cancel{(x-2)}}{2x} \cdot \frac{1}{\cancel{x-2}}$$

$$= \lim_{x\to 2} \frac{-1}{2x} = \frac{-1}{4} \text{ so } \boxed{f'(2) = -\frac{1}{4}}$$

b)

$$K'(-2) = \lim_{x\to -2} \frac{(3-x^2)-(3-(-2)^2)}{x+2}$$

$$= \lim_{x\to -2} \frac{\cancel{3}-x^2-\cancel{3}+4}{x+2} = \lim_{x\to -2} \frac{-x^2+4}{x+2}$$

$$= \lim_{x\to -2} \frac{-(x^2-4)}{x+2} = \lim_{x\to -2} \frac{-(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}$$

$$= \lim_{x\to -2} -(x-2) = -(-2-2) = 4$$

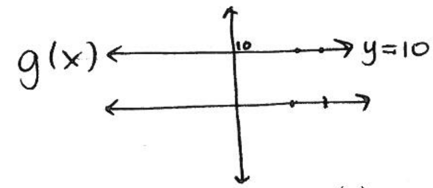
$\boxed{\text{SO } K'(-2) = 4}$

c)

$$g'(3) = \lim_{x\to 3} \frac{g(x)-g(3)}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3}$$

$$= \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{(x+1)-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$= \frac{1}{\sqrt{3+1}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4} = g'(3)}$$



5. Consider the function $g(x) = 10$.

a) Using what you know about the graph of $g(x)$, what does $g'(6) = ?$

$g'(6) = 0$ because @ $x=6$, slope $=0$.

b) Use the alternative definition AND the original definition of the derivative to verify your answer for $g'(6)$.
YES … use BOTH definitions ☺

original

$$g'(6) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{10 - 10}{h}$$

$$= \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0$$

alternate

$$g'(6) = \lim_{x \to 6} \frac{g(x) - g(6)}{x-6}$$

$$= \lim_{x \to 6} \frac{10-10}{x-6}$$

$$= \lim_{x \to 6} \frac{0}{x-6} = 0$$

6. If $f(2) = 3$ and $f'(2) = 5$, find an equation of the tangent line at the point where $x = 2$.

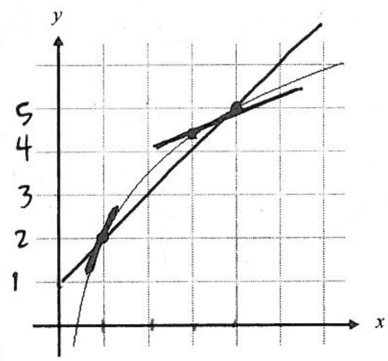
point (2,3)

slope of $f(x)$ at $x=2$ is 5

$x=2$
$y=3$
$m=5$

$y - 3 = 5(x-2)$

7. Use the figure to the right to answer the following questions:



a) What is $f(1)$ and $f(4)$?

$f(1) = 2$

$f(4) = 5$

b) What is the geometric interpretation of $\frac{f(4) - f(1)}{4-1}$?

$\frac{y_2 - y_1}{x_2 - x_1}$ Slope (slope of secant line between (1,2) & (4,5))

c) Using the geometric interpretation of each expression, insert the proper inequality symbol (< or >).

i) $\frac{f(4) - f(1)}{4-1}$ $\boxed{>}$ $\frac{f(4) - f(3)}{4-3}$

steeper positive slope

ii) $\frac{f(4) - f(1)}{4-1}$ $\boxed{<}$ $f'(1)$

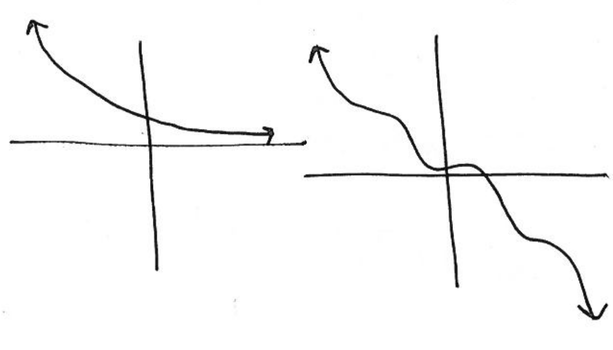
slope of secant between $x=1$, $x=4$

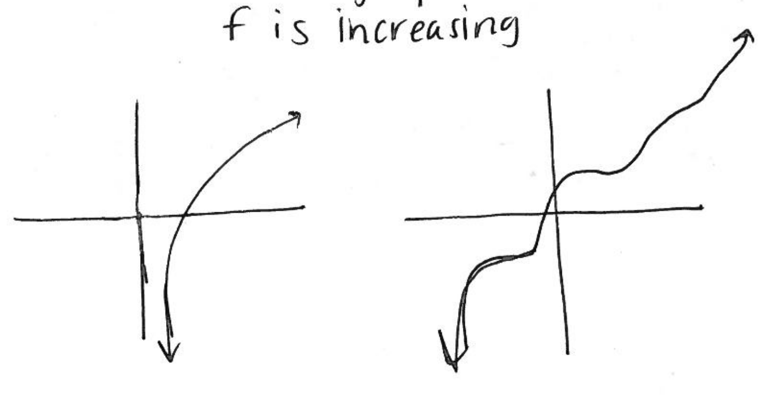
slope of tangent line @ $x=1$

8. Sketch a function whose derivative is ALWAYS negative and another whose derivative is ALWAYS positive.

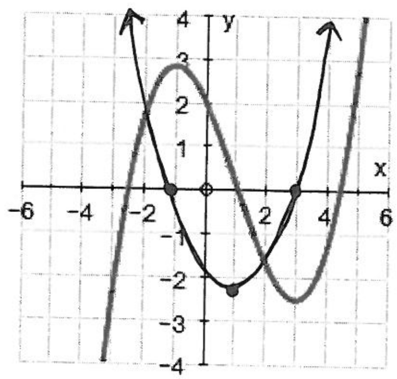
f' is always negative when f is decreasing

f' is always positive when f is increasing





9. Use the graph of $f(x)$ shown to the right.



a) Where is $f'(x)=0$? Explain.

$f'(x)=0$ when $x=^{-}1$ and $x=3$

b) Where is $f'(x)>0$? Explain.

$f'(x)>0$ when $f(x)$ is increasing

c) Where is $f'(x)<0$? Explain.

$f'(x)<0$ when $f(x)$ is decreasing.

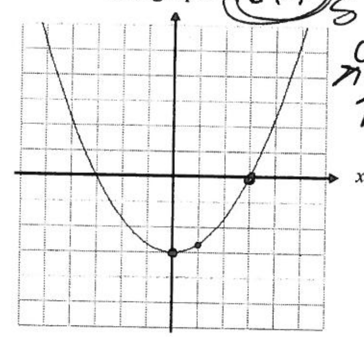
d) On the same graph, draw a possible sketch of $f'(x)$.

See graph

10. The figure to the right shows the graph of $g'(x)$.

The graph of $g'(x)$

Slopes of $g(x)$



a) What does $g'(0)=$? ... How about $g'(3)$?

$g'(0)=-3$ slope @ $x=0$ is -3.

$g'(3)=0$ Slope @ $x=3$ is 0.

c) From the graph, $g'(1)=-\frac{8}{3}$. What does this tell us about the graph of g?

at $x=1$, the slope of $g(x)$ is $-\frac{8}{3}$

d) From the graph, $g'(4)=\frac{7}{3}$. What does this tell us about the graph of g?

at $x=4$, the slope of $g(x)$ is $\frac{7}{3}$.

e) Is $g(6)-g(4)$ positive or negative (those are g values not g')? Explain.

change in y-values between $g(4)$ and $g(6)$ is positive,
Because g' is positive so g is increasing between 4 & 6

f) Find (if they exist) any value(s) of x, where $g'(x)=0$?

$g'(x)=0$ when $x=-3$, $x=3$ (look at graph)

g) Is it possible to find $g(2)$ from this graph? Explain.

NO. we need to know some original value of $g(x)$

h) What interval is $g(x)$ increasing? What interval is $g(x)$ decreasing? How do you know?

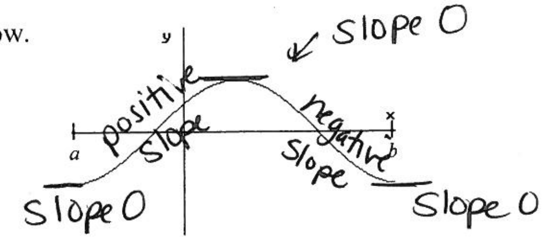
$g(x)$ is increasing when $g'(x)>0$. This is when x is $(-\infty,-3)\cup(3,\infty)$

$g(x)$ is decreasing when $g'(x)<0$. This is when x is $(-3,3)$

i) If you were told that $g(2)=1$, sketch a possible graph of $g(x)$?

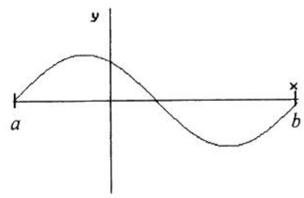
See above.

11. The graph of f is shown below.

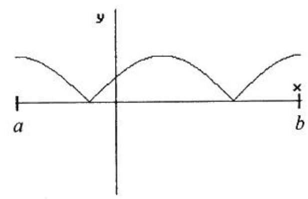


Which of the following could be the graph of the derivative of f?

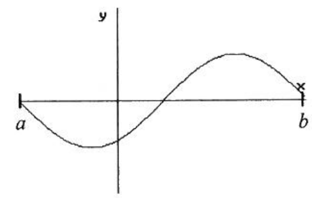
A.



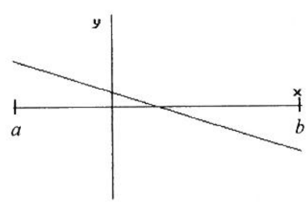
~~B.~~



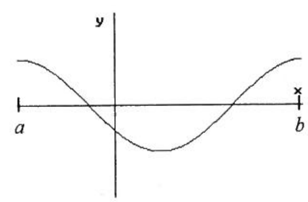
C.



~~D.~~

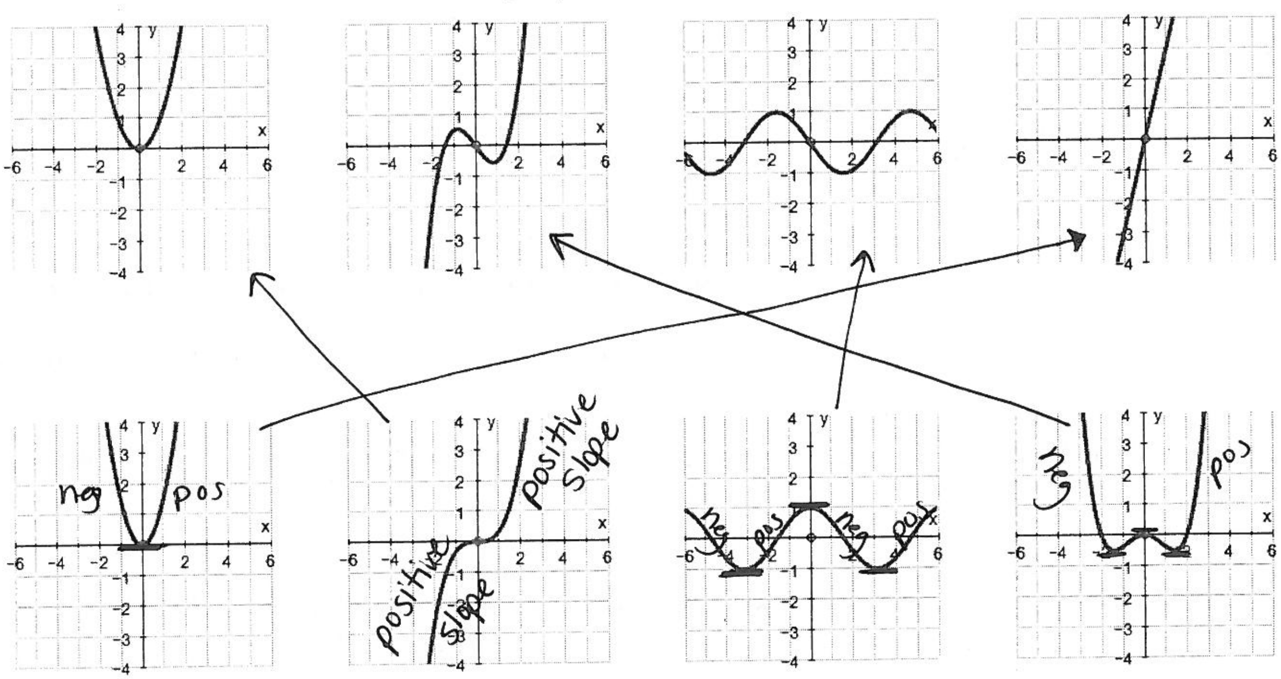


~~E.~~



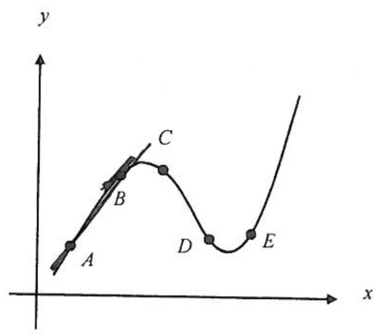
12. The graphs in the first row are the derivatives. Match them with the graph of their function shown in the second row.

(Graphs of Derivative)



(Graphs of Function)

13. Use the graph of f below to answer each question.



a) Between which two consecutive points is the average rate of change of the function greatest?

A & B

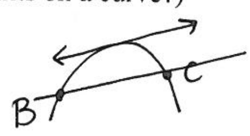
b) Is the average rate of change between A and B greater than or less than the instantaneous rate of change of B?

Greater than? (It's close)

c) Give any sets of consecutive points for which the average rates of change of the function are approximately equal.

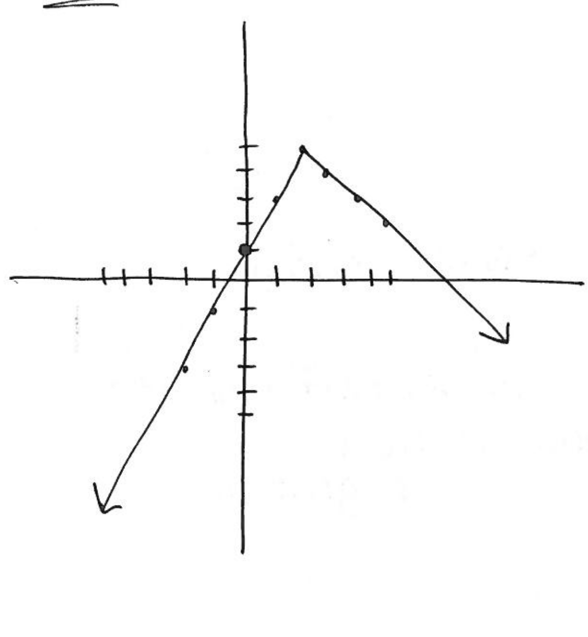
B & C & D & E

d) Sketch a tangent line to the graph somewhere <u>between</u> the points B and C such that the slope of the tangent line you draw is the same as the average rate of change of the function between B and C. (Do you think it would be possible to do this for ANY two points on a curve?)



it's possible to do this for any points on a curve if fctn is continuous.

14. Sketch the graph of a <u>continuous</u> function f with $f(0) = 1$ and $f'(x) = \begin{cases} 2 & \text{if } x < 2 \\ -1 & \text{if } x > 2 \end{cases}$



AP Calculus
3.2 Worksheet

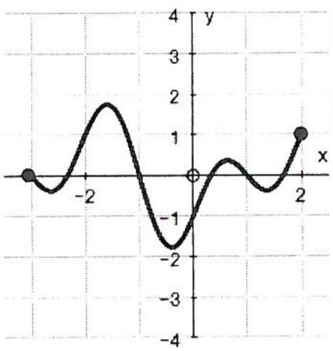
All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. When does a derivative fail to exist?

For questions 2 – 4, the graph of a function over a closed interval D is given. At what domain points does the function appear to be

a) differentiable? b) continuous but not differentiable c) neither continuous or differentiable?

2. $D: \ -3 \le x \le 2$

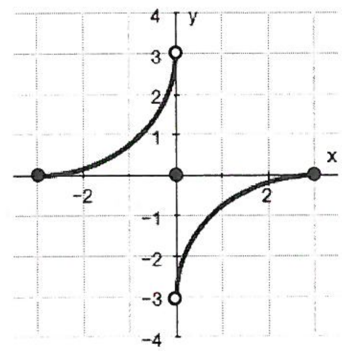


a) $(-3,3)$

b) none

c) none

3. $D: \ -3 \le x \le 3$

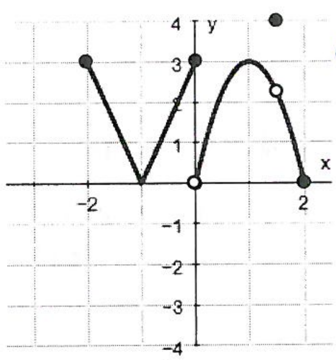


a) $(-3,0) \cup (0,3)$

b) none

c) $x=0$

4. $D: \ -2 \le x \le 2$

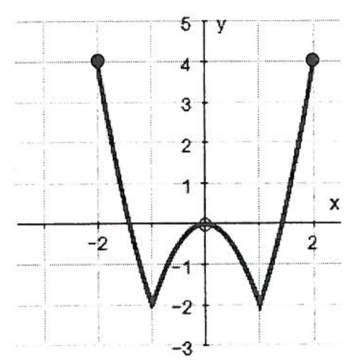


a) $(-2,-1) \cup (-1,0)$
$\cup (0,1.5) \cup (1.5,2)$

b) $x=-1$

c) $x=0, x=1.5$

5. $D: \ -2 \le x \le 2$



a) $(-2,-1) \cup (-1,1)$
$\cup (1,2)$

b) $x=-1, x=1$

c) none

6. The graph of the function f shown in the figure below has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, –1) and (3, 1). For what values of x, $-2 < x < 4$, is f not differentiable?

A) 0 only

B) 0 and 2 only

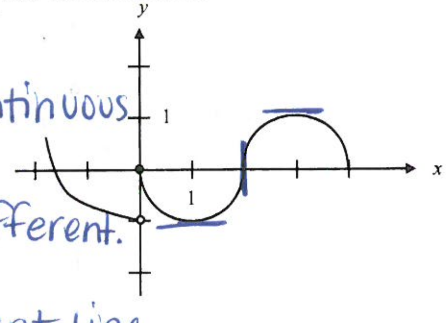
C) 1 and 3 only

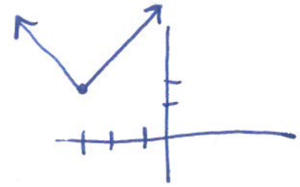
D) 0, 1, and 3 only

E) 0, 1, 2, and 3

$x=0$ ... f is not continuous

$x=2$ ... f is not different.
b/c. vertical
tangent line





7. Suppose $f(x) = 2 + |x+3|$.

a) What is the value of $f'(3)$? Explain your answer.

$f'(3) = 1$ because slope @ $x=3$ is 1.

b) What is the value of $f'(-3)$? Explain your answer.

$f'(-3)$ DNE because when $x=-3$, $f'(x)$ is undefined (corner)

8. What are the three different derivative "formulas" ?... (don't forget to use a limit)

original	alternate	symmetric
$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$	$\lim_{x \to c} \frac{f(x)-f(c)}{x-c}$	$\lim_{h \to 0} \frac{f(x+h)-f(x-h)}{2h}$

9. If f is a function such that $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2} = 0$, which of the following must be true? → means $f'(2)=0$

A) The limit of $f(x)$ as x approaches 2 does not exist. No
B) f is not defined at $x = 2$. No
C) The derivative of f at $x = 2$ is 0. ✓ yes
D) f is continuous at $x = 0$. not $x=2$
E) $f(2) = 0$ No

10. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true? → means $f'(2)=5$

I. f is continuous at $x = 2$. yes
II. f is differentiable at $x = 2$. yes
III. The derivative of f is continuous at $x = 2$. not sure?

A) I only B) II only C) I and II only D) I and III only E) II and III only

11. Let f be a function that is differentiable on the open interval (0, 10). If $f(2) = -5$, and $f(5) = 5$, and $f(9) = -5$, each of the following statements MUST be true. Explain why each statement must be true.

a) f has at least 2 zeros. If $f(x)$ is differentiable, then $f(x)$ is continuous

b) The graph of f has at least one horizontal tangent line.

c) For some c, $2 < c < 5$, $f(c) = 3$.

12. The function f is defined on the closed interval [0, 8]. The graph of its derivative f' is shown below. The point (3, 5) is on the graph of $f(x)$. An equation of the tangent line to the graph of f at (3, 5) is

A) $y = 2$

B) $y = 5$

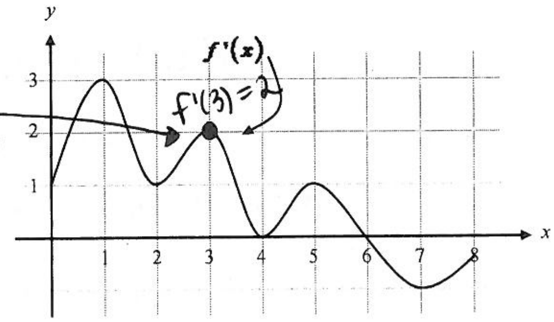
C) $y - 5 = 2(x-3)$

D) $y + 5 = 2(x-3)$

E) $y + 5 = 2(x+3)$

$x = 3$
$y = 5$
$m = 2$ (from graph)

$y - 5 = 2(x-3)$



13. Let $g(x) = \begin{cases} 3x-2 & \text{if } x \le 0 \\ x^2 - 1 & \text{if } x > 0 \end{cases}$. Which of the following is equal to the left-hand derivative of g at $x = 0$?

left hand slope at $X=0$?

A) $2x$ B) 3 C) 0 D) ∞ E) $-\infty$

$3X-2$ is graph
3 is slope.

14. Suppose $f(x) = \begin{cases} 3-x & \text{if } x < 1 \\ mx^2 + nx & \text{if } x \ge 1 \end{cases}$

a) If the function is continuous, what is the relationship between m and n. (Use the definition of continuity!)

$3 - 1 = 2$

$2 = m(1)^2 + n(1)$

$2 = m + n$

$\boxed{2 = m + n}$

b) What is the derivative of the portion of the graph where $x < 1$.

graph is $3-x$, slope is -1 $f'(x) = -1$ for $x<1$

c) Using whatever method you wish to show/explain, find the derivative of the portion of the graph where $x \ge 1$.

$x \ge 1$, $mx^2 + nx$

derivative is $2m + n$

d) In order for $f(x)$ to be differentiable at $x = 1$, what is the relationship between the answers in part b and c?

left hand der. = right hand der.

$-1 = 2m + n$

e) Using your answers from part (d) and from part (a), solve for m and n.

continuous $2 = m + n$

differentiable $(-1 = 2m + n)\ -1$

$2 = m + n$
$1 = -2m - n$

$3 = -m$

$\boxed{-3 = m}$

$2 = m + n$
$2 = -3 + n$
$\boxed{5 = n}$

One of the four (4) required calculator skills on the AP exam is for you to take a derivative at a point. Use your calculator to answer the following questions. Be sure to use correct mathematical notation.

15. Using your calculator, find the equation of the tangent line to the graph of $f(x) = x^3 + x^2$ when $x = 2$. Show your work using correct notation.

$x = 2$ $\quad$ nDeriv $(x^3+x^2, x, 2)$

$y = 2^3+2^2 = 8+4 = 12$ $\quad$ $\frac{d}{dx}(x^3+x^2)\Big|_{x=2} = 16$

$m = 16$

$\boxed{y-12 = 16(x-2)}$

16. When an object falls its distance traveled (in meters) can be modeled by the equation $h(t) = 4.9t^2$. The derivative of h with respect to t is the velocity of the object. Find the velocity of the object at $t = 3$ seconds.

$\frac{dh}{dt}$ = nDeriv $(4.9x^2, x, 3)$

$= \frac{d}{dx}(4.9x^2)\Big|_{x=3}$ $\quad$ $= 29.4$ m/sec.

17. Suppose $f(x) = |4 - x^2|$.

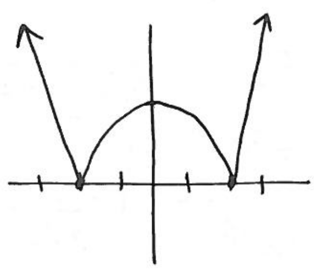
a) Find the slope of the function when $x = 3$.

nDeriv $\quad f'(3) = 6$

b) Find the slope of the function when $x = 2$.

nDeriv $\quad f'(2) = .001$ $\quad$ but should $f'(x)$ exist at $x=2$?

c) Graph the function and explain any issues with your answer from part (b).



$f'(2)$ DNE because

$$\underbrace{\lim_{x \to 2^-} f'(x) \neq \lim_{x \to 2^+} f'(x)}_{\text{algebraic way to say "not smooth"}}$$

AP Calculus
3.3 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. A derivative tells you the __Slope__ of a function. Slope at a point, instantaneous rate of change of fctn.

2. What is the power rule for derivatives? (i.e. how do you take the derivative of $y = x^n$?)

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

power becomes coefficient then reduce power by 1.

3. For each of the following functions, find $\frac{dy}{dx}$.

a) $y = -2x^3 + x$

$\boxed{y' = -6x^2 + 1}$

b) $y = \frac{x^4}{3} - \frac{x^2}{7} + 5$

$\boxed{y' = \frac{4}{3}x^3 - \frac{2}{7}x}$

c) $y = \frac{5}{x^2} + \frac{6}{x} - 8x^3$

$\boxed{y' = -10x^{-3} - 6x^{-2} - 24x^2 \text{ or } = \frac{-10}{x^3} - \frac{6}{x^2} - 24x^2}$

d) $y = \frac{x^{-3}}{2} + 5x^{-4} - 3x^{-6}$

$\boxed{y' = -\frac{3}{2}x^{-4} - 20x^{-5} + 18x^{-7}}$

e) $y = 5x^4 + 2x^3 - 8x^2 - 7x + 11$

$\boxed{y' = 20x^3 + 6x^2 - 16x - 7}$

f) $y = 7x - 8$

$\boxed{y' = 7}$

g) $y = (x^2 - 3)(x + 4)$

FOIL

$y = x^3 + 4x^2 - 3x - 12$

$\boxed{y' = 3x^2 + 8x - 3}$

h) $y = \frac{x^5 - 2x^4 + 3x^3}{x^5}$

$= \frac{x^5}{x^5} - \frac{2x^4}{x^5} + \frac{3x^3}{x^5}$

$= 1 - \frac{2}{x} + \frac{3}{x^2}$

$= 1 - 2x^{-1} + 3x^{-2}$

$\boxed{y' = 2x^{-2} - 6x^{-3}}$

i) $y = \sqrt{x} + \frac{3}{\sqrt{x}} - 6x^{5/3} + \frac{7}{x^3}$

$y = x^{1/2} + 3x^{-1/2} - 6x^{5/3} + 7x^{-3}$

$\boxed{y' = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{-3/2} - 10x^{2/3} - 21x^{-4} = \frac{1}{2\sqrt{x}} - \frac{3}{2\sqrt{x^3}} - 10x^{2/3} - \frac{21}{x^4}}$

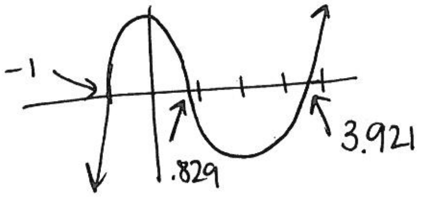
4. [Calculator Required] We want to find all points where the graph of $y = x^4 - 5x^3 - 3x^2 + 13x + 10$ has a horizontal tangent line.

a) First, find an equation for y' .

b) A horizontal tangent line will have a slope = __0__. So set $y' =$ __0__, and use your calculator to solve this equation.

$4x^3 - 15x^2 - 6x + 13 = 0$

graph & find zeros.



find all pts of y= where $y' = 0$

(−1, 0)
(.829, 16.339)
(3.921, −50.194)

y-values of y= graph

$x = 1$
$y = 0/2 = 0$
$m = 3/2$

5. Find the equation of the tangent line to the function $y = \dfrac{x^2 + x - 2}{2x}$ at the point where $x = 1$.

$$y = \frac{x^2}{2x} + \frac{x}{2x} - \frac{2}{2x}$$

$$= \frac{x}{2} + \frac{1}{2} - \frac{2}{2x} = \frac{1}{2}x + \frac{1}{2} - x^{-1}$$

$$y' = \frac{1}{2} + x^{-2}\Big|_{x=1}$$

$$= \frac{1}{2} + \frac{1}{(1)^2} = \frac{1}{2} + 1 = \frac{3}{2} \text{ slope @ } x = 1$$

$$\boxed{y - 0 = \frac{3}{2}(x-1)}$$

6. Find the equation of the normal line to the function $y = x^3 - 5x + 1$ at the point when $x = 2$.

$x = 2$
$y = -1$
$m = -1/7$

$$3x^2 - 5\Big|_{x=2}$$

$$3(2)^2 - 5 = 12 - 5 = 7 \text{ normal is } \perp \rightarrow -1/7$$

$$\boxed{y + 1 = -1/7(x-2)}$$

7. Find the points on the curve $y = x^3 + 3x^2 - 9x + 7$ where the tangent line is parallel to the x-axis.

slope = 0

$y' = 3x^2 + 6x - 9$
$0 = 3x^2 + 6x - 9$
$0 = 3(x^2 + 2x - 3)$
$\rightarrow 0 = 3(x+3)(x-1)$
$x = -3 \quad x = 1$

tangent line is parallel to x-axis ~~when~~ at $(-3, 34)$ and $(1, 2)$

8. Consider the curve $y = x^3 + x$.

a) Find the tangents to the curve at all the points where the slope is 4. (be careful! … it doesn't say $x = 4$!)

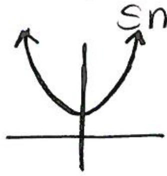
$f'(x) = 4 \rightarrow$ tangent lines

$y' = 3x^2 + 1$
$4 = 3x^2 + 1$
$3 = 3x^2$
$\rightarrow 1 = x^2$
$x = 1, -1$

$x = 1$
$y = 2$
$m = 4$
$\boxed{y - 2 = 4(x-1)}$

$x = -1$
$y = -2$
$m = 4$
$\boxed{y + 2 = 4(x+1)}$

b) What is the smallest slope of the curve? At what value of x does the curve have this value?



smallest slope of curve will occur when derivative graph is at a minimum.

at $x = 0$ $f'(x)$ has a minimum (1)

at $x = 0$, smallest slope is 1.

9. Find the x- and y-intercepts of the line that is tangent to the curve $y = x^3$ at the point $(-2, -8)$.

$y' = 3x^2$
$y'(-2) = 3(-2)^2 = 12$
Slope of $y = x^3$ at $x = -2$ is 12.

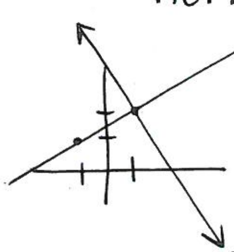
tangent line $y + 8 = 12(x+2)$
$y + 8 = 12x + 24$
$y = 12x + 16$

x-int ($y=0$)	y-int ($x=0$)
$0 = 12x + 16$	
x-int $= -4/3$	y-int $= 16$

10. If the line normal to the graph of f at the point (1, 2) passes through the point (–1, 1), then which of the following gives the value of $f'(1)$?

normal slope is $\perp$ to tangent slope (which is f')

A	–2
B	2
C	–1/2
D	1/2
E	3



normal slope $\frac{1}{2}$

tangent slope $\perp$ to $\frac{1}{2} \rightarrow -2$

To be differentiable at a point, the left and right derivatives must be equal at that point (see last WS). Use this concept and the <u>definition</u> of continuity to solve for the parameters (those are those pesky little letters) in the next question.

11. Solve for a and b in order for $g(x)$ to be both continuous and differentiable at $x = 0$. (look back at the 3.2 WS if you need help)

$$g(x)=\begin{cases} ax+b & ;x>0 \\ 1-x+x^2 & ;x\leq 0 \end{cases}$$

~~12.~~ When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

A) 3
B) 4
C) 6
D) 8
E) 12

~~13.~~ Let $f(x)=\sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

A) $\frac{1}{4}$
B) 1
C) 4
D) $\frac{1}{\sqrt{2}}$
E) $\frac{1}{2\sqrt{2}}$

~~14.~~ [Calculator] Which of the following is an equation of the tangent line to $f(x)=x^4+2x^2$ at the point where $f'(x)=1$?

A) $y = 8x - 5$
B) $y = x + 7$
C) $y = x + .763$
D) $y = x - .122$
E) $y = x - 2.146$

AP Calculus
3.3 Worksheet Day 2

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the product rule? $\frac{d}{dx}(uv) = u \cdot v' + v \cdot u'$

2. What is the quotient rule? $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

3. Let $f(x) = (3x^3 + 4x^2)(2x^4 - 5x)$.

a) Find $f'(x)$ without using the product rule

$f(x) = 6x^7 - 15x^4 + 8x^6 - 20x^3$

$f'(x) = 42x^6 - 60x^3 + 48x^5 - 60x^2$ ✓

b) Find $f'(x)$ using the product rule.

$f'(x) = (3x^3+4x^2)(8x^3-5) + (2x^4-5x)(9x^2+8x)$

$= 24x^6 - 15x^3 + 32x^5 - 20x^2 + 18x^6 + 16x^5 - 45x^3 - 40x^2$

$= 42x^6 + 48x^5 - 60x^3 - 60x^2$ ✓

4. Let $f(x) = \frac{x^2+4}{x}$.

a) Find $f'(x)$ without using the quotient rule

$f(x) = \frac{x^2}{x} + \frac{4}{x}$

$= x + 4x^{-1}$

$f'(x) = 1 - 4x^{-2}$

$= 1 - \frac{4}{x^2}$ ✓

b) Find $f'(x)$ using the quotient rule.

$f'(x) = \frac{x(2x) - (x^2+4)(1)}{x^2}$

$= \frac{2x^2 - x^2 - 4}{x^2}$

$= \frac{x^2}{x^2} - \frac{4}{x^2} = 1 - \frac{4}{x^2}$ ✓

5. Find $\frac{dy}{dx}$ for each of the following functions.

a) $y = \frac{2x-5}{3x+2}$

$\frac{dy}{dx} = \frac{(3x+2)(2) - (2x-5)(3)}{(3x+2)^2}$

$= \frac{6x + 4 - 6x + 15}{(3x+2)^2}$

$= \boxed{\frac{19}{(3x+2)^2}}$

b) $y = (3-x)(2+x^2)^{-1}$

$= \frac{3-x}{2+x^2}$

$\boxed{y' = \frac{(2+x^2)(-1) - (3-x)(2x)}{(2+x^2)^2}}$

$= \frac{-2 - x^2 - 6x + 2x^2}{(2+x^2)^2}$

$= \boxed{\frac{x^2 - 6x - 2}{(2+x^2)^2}}$

c) $y = \frac{x^3}{8-x^2}$

$y' = \frac{(8-x^2)(3x^2) - (x^3)(-2x)}{(8-x^2)^2}$

$= \frac{24x^2 - 3x^4 + 2x^4}{(8-x^2)^2}$

$= \boxed{\frac{24x^2 - x^4}{(8-x^2)^2}}$

6. For $a - d$, write a expression for $f'(x)$ and then use it to find $f'(2)$ given the following information:

$$g(2)=3 \quad g'(2)=-2$$
$$h(2)=-1 \quad h'(2)=4$$

a) $f(x)=2g(x)+h(x)$

$f'(x) = 2\cdot g'(x) + h'(x)$

$f'(+2) = 2(-2) + (4)$

$= -4 + 4 = \boxed{0}$

b) $f(x)=4-h(x)$

$f'(x) = -h'(x)$

$f'(2) = -h'(2)$

$= \boxed{4}$

c) $f(x)=g(x)h(x)$

$f'(x) = g(x)\cdot h'(x) + h(x)\cdot g'(x)$

$f'(2) = g(2)\cdot h'(2) + h(2)\cdot g'(2)$

$= 3\cdot 4 + -1\cdot -2$

$= 12 + 2 = \boxed{14}$

d) $f(x)=\dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)\cdot g'(x) - g(x)\cdot h'(x)}{h^2(x)}$

$f'(2) = \dfrac{h(2)\cdot g'(2) - g(2)\cdot h'(2)}{h^2(2)}$

$= \dfrac{(-1)(-2) - (3)(4)}{(-1)^2} = \boxed{-10}$

7. Suppose u and v are differentiable functions of ~~[illegible]~~ x and that $u(3) = 4$, $\left.\dfrac{du}{dx}\right|_{x=3} = -3$, $v(3) = 2$, and $\left.\dfrac{dv}{dx}\right|_{x=3} = 3$. Find the values of the following derivatives at $x = 3$.

$u'(3) = -3$ $\quad$ $v'(3) = 3$

a) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\cdot u' - u\cdot v'}{v^2}$

$= \dfrac{2\cdot(-3) - 4(3)}{2^2}$

$= -\dfrac{18}{4} = \boxed{-\dfrac{9}{2}}$

b) $\dfrac{d}{dx}(uv) = u\cdot v' + v\cdot u'$

$= 4\cdot 3 + 2\cdot -3$

$= 12 - 6$

$= \boxed{6}$

c) $\dfrac{d}{dx}(5u - 2v + 4uv)$

$\dfrac{d}{dx} = 5u' - 2v' + 4u\cdot v' + v\cdot 4u'$

$= 5(-3) - 2(3) + 4(4)(3) + 2(4)(-3)$

$= -15 - 6 + 36 - 24 = \boxed{3}$

d) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right) = \dfrac{uv' - vu'}{u^2}$

$= \dfrac{4(3) - (2)(-3)}{4^2} = \dfrac{18}{16} = \boxed{\dfrac{9}{8}}$

8. Solve for a and b in order for $f(x)$ to be both continuous and differentiable at $x = 1$. (be sure to use the definition of continuity)

$$f(x) = \begin{cases} x^2 + 2 & ;x \le 1 \\ a\left(x - \frac{1}{x}\right) + b & ;x > 1 \end{cases}$$

9. For each of the following, find the equation of the tangent line to the given function at the indicated point.

a) $f(x)=(x^3-3x+1)(x+2)$ at the point (1, –3).

$$f'(x)=(x^3-3x+1)(1)+(x+2)(3x^2-3)\Big|_{x=1}$$
$$=(1^3-3(1)+1)(1)+(1+2)(3(1)^2-3)$$
$$=(1-3+1)+(3)(0)=-1+0=-1$$

$x=1$
$y=-3$
$m=-1$

$$\boxed{y+3=-1(x-1)}$$

b) $y=\dfrac{8}{4+x^2}$ at the point (–2, 1).

$$y'=\frac{(4+x^2)(0)-8(2x)}{(4+x^2)^2}\Bigg|_{x=-2}$$
$$=\frac{0-8(2(-2))}{(4+(-2)^2)^2}=\frac{32}{64}=\frac{1}{2}$$

$x=-2$
$y=1$
$m=\frac{1}{2}$

$$\boxed{y-1=\tfrac{1}{2}(x+2)}$$

10. At what point on the graph of $y=\frac{1}{2}x^2$ is the tangent line parallel to the line $2x-4y=3$?

A) $\left(\frac{1}{2},\frac{1}{2}\right)$
B) $\left(\frac{1}{2},\frac{1}{8}\right)$
C) ~~$\left(1,-\frac{1}{4}\right)$~~
D) ~~$\left(1,\frac{1}{2}\right)$~~
E) ~~$(2,2)$~~

$y=\frac{1}{2}x^2$
$y'=1x$
$y'=x$

$\frac{-4y}{-4}=\frac{3-2x}{-4}$
$y=-\frac{3}{4}+\frac{1}{2}x$ Slope is ½

when is derivative parallel to line w/ slope ½

$\frac{1}{2}=x$ $\quad$ $x=\frac{1}{2}$ $\quad$ when $x=\frac{1}{2}$

$y=\frac{1}{2}\left(\frac{1}{2}\right)^2=\frac{1}{2}\left(\frac{1}{4}\right)=\frac{1}{8}$

11. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

A) 0.4
B) 0.5
C) 2.6
D) 3.4
E) 5.5

$x=3$
$y=2$
$m=5$

$y-2=5(x-3)$
$y=5(x-3)+2$ approximate a zero

$0=5(x-3)+2$
$\frac{-2}{5}=\frac{5(x-3)}{5}$
$-2/5=x-3$
$+3 \quad\quad +3$

$x=3-2/5$
$=2\,3/5=2.6$

12. An equation of the line tangent to the graph of $y=\dfrac{2x+3}{3x-2}$ at the point (1, 5) is

A) ~~$13x-y=8$~~
B) $13x+y=18$
C) ~~$x-13y=64$~~
D) $x+13y=66$
E) $-2x+3y=13$

$$y'=\frac{(3x-2)(2)-(2x+3)(3)}{(3x-2)^2}\Bigg|_{x=1}$$

$$=\frac{(3-2)(2)-(2+3)(3)}{(1)^2}=\frac{2-15}{1}=\frac{-13}{1}=-13$$

$x=1$
$y=5$
$m=-13$

$y-5=-13(x-1)$
$y-5=-13x+13$
$y=-13x+18$

13. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

A) -2
B) $\frac{1}{6}$
C) $\frac{1}{2}$
D) 2
E) 6

$$f'(x) = \left.\frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2}\right|_{x=2}$$

$$= \frac{(2-1)(2\cdot 2) - (2^2-2)(1)}{(2-1)^2} = \frac{4-2}{1^2} = \frac{2}{1} = 2$$

14. If u, v, and w are nonzero differentiable functions of x, then the $\frac{d}{dx}\left(\frac{uv}{w}\right)$ is

Quotient rule

do find $(uv)'$, use product rule

A) $\frac{uv' + u'v}{w'}$

B) $\frac{u'v'w - uvw'}{w^2}$

C) $\frac{uvw' - uv'w - u'vw}{w^2}$

D) $\frac{u'vw + uv'w + uvw'}{w^2}$

E) $\frac{uv'w + u'vw - uvw'}{w^2}$

$$\frac{w(uv)' - (uv)w'}{w^2}$$

$$\frac{w(uv' + vu') - uvw'}{w^2}$$

$$\frac{wuv' + wvu' - uvw'}{w^2}$$

15. When an object is thrown off a 100 foot cliff with an initial velocity of 40 feet/second, the height h, in feet, of the object can be modeled as a function of time t, in seconds, using the function

$$h(t) = -16t^2 + 45t + 100. \quad \text{feet}$$

a) Find $\frac{dh}{dt}$ … What is the unit of measurement for this equation?

$-32t + 45$ $\qquad \frac{\Delta h}{\Delta t} = \frac{\text{feet}}{\text{sec}}$

b) Find $\frac{d^2h}{dt^2}$ … What is the unit of measurement for this equation?

-32 $\qquad \frac{\text{ft/sec}}{\text{sec}}$ or ft/sec^2

16. Let $g(x) = x - \frac{1}{x}$. Find the following:

$= x - x^{-1}$

a) $g'(x)$

$= 1 + x^{-2}$

b) $g''(x)$

$-2x^{-3}$

c) The tangent line equation when $x = 2$

$g'(2) = 1 + x^{-2}\big|_2$

$= 1 + 2^{-2} = 1 + \frac{1}{2^2}$

$= 1 + \frac{1}{4}$

$= 5/4$

$x = 2$
$y = \frac{3}{2}$
$m = 5/4$

$\boxed{y - \frac{3}{2} = 5/4(x - 2)}$

AP Calculus
3.4 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the relationship between position, velocity, and acceleration?

If Position is $s(t)$, then velocity is derivative of position $(v(t)=s'(t))$ and acceleration is derivative of velocity $(a(t)=v'(t)=s''(t))$

2. Once again trying to blow up earth because it interferes with his view of Venus, Marvin the Martian lands on the moon. Bugs Bunny, as always, interferes with his plan. Chasing Bugs, Marvin fires a warning shot straight up into the air with his Acme Disintegration Pistol. The height (in feet) after t seconds of the shot is given by

$$s(t) = -2.66t^2 + 135t + 3.$$

a) Find the velocity and acceleration as functions of time. (What is the meaning of the acceleration function?)

$v(t)=s'(t)=-5.32t+135$

$a(t)=v'(t)=s''(t)=-5.32$ acceleration means the pull of gravity of the moon is $-5.32\ ft/sec^2$.

b) What is the position of the shot when the velocity is 0?

$v(t)=0=-5.32t+135$
$-135=-5.32t$
$25.3759398\ldots=t$
Velocity is 0 when $t=25.375\ldots$

what is position at $t=25.375\ldots$?
$s(25.3759398) \approx \boxed{1712.876 \text{ ft above moon}}$
don't round

3. Fill in the blanks.

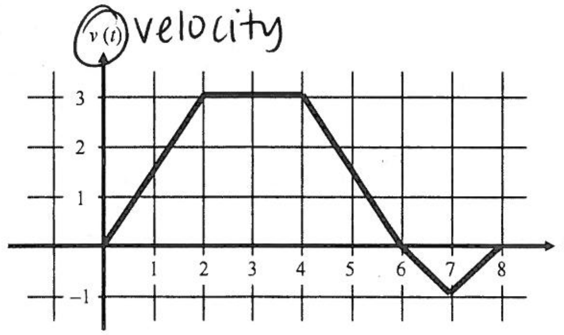
a) When the velocity is positive, the object is moving in a positive direction.

b) An object is slowing down when the velocity and acceleration have different signs.

c) An object is stopped when velocity/speed is zero.

d) Speed is always positive because it is the absolute value of velocity.

4. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity, v, of the bug at time t, $0 \le t \le 8$ is given by the function whose graph is shown below.



a) At what value of t does the bug change direction? Justify your response.

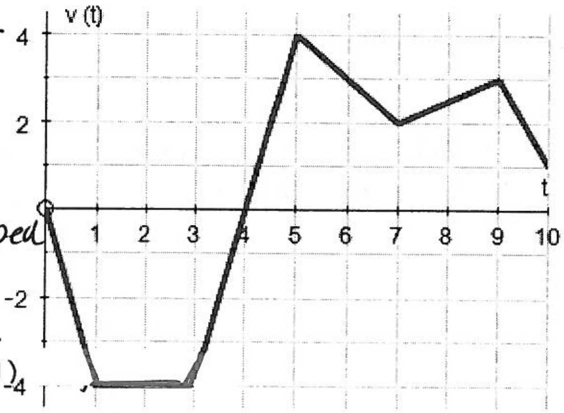
Bug changes direction at $t=6$. That is where velocity changes sign.

b) During which time intervals in the bug slowing down? Justify your response.

The bug is slowing down when velocity and acceleration have opposite signs. On $[4,6]$ $v(t)>0$ and $a(t)<0$, so bug is slowing
On $[7,8]$ $v(t)<0$ and $a(t)>0$, so bug is slowing.

horizontal

5. The figure graphed below shows the velocity of a particle moving along a coordinate line. Justify each response.



a) When is the particle moving right?

when $v(t)>0$, particle moves right
this happens on $(4,10)$

b) When is the particle moving left?

when $v(t)<0$, particle moves left.
This happens on $(0,4)$

c) When is the particle stopped?

when $v(t)=0$, the particle is stopped
This happens at $t=4$

d) When is the particle speeding up?

Particle speeds up when $v(t)$ and $a(t)$ have same signs.
$v(t)>0$ & $a(t)>0$ on $(4,5)\cup(7,9)$. $v(t)<0$ & $a(t)<0$ on $(0,1)$

e) When does the particle change directions?

Particle changes direction when $v(t)$ changes sign.
at $t=4$ $v(t)$ changes from $-$ to $+$.

f) When is the particle slowing down?

Particle slows down when $a(t)$ and $v(t)$ have opp. signs.
$v(t)>0$ & $a(t)<0$ on $(5,7)\cup(9,10)$ $v(t)<0$ and $a(t)>0$ on $(3,4)$

g) ~~What~~ when is the particle moving at its greatest speed?

speed = |velocity| = 4 and occurs
$(1,3)\cup(5)$.

h) When is the particle's acceleration positive?

Particle's acceleration is positive when
slope of $v(t)>0$. This occurs $(3,5)\cup(7,9)$

i) When is the particle's acceleration negative?

Particles acceleration is negative
when slope of $v(t)<0$. This occurs
$(0,1)\cup(5,7)\cup(9,10)$

6. Fill in the blanks with correct mathematical notation.

a) If you want the average velocity of a particle on the interval [2, 5], you must find Slope $\frac{v(5)-v(2)}{5-2}$.

b) If you want the velocity of a particle at $t = 4$, you must find $v(4)$ or $s'(4)$.

7. Velocity is the rate of change of position. If the position of a particle on the x – axis at time t is given by $-5t^2$, then what is the average velocity of the particle for $0 \le t \le 3$?

avg velocity = slope of position $\frac{s(3)-s(0)}{3-0} = \frac{-45-0}{3-0} = \frac{-45}{3} = -15$ units/sec

$s(t) = -5t^2$

8. A particle moves along the x – axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero? At what instant is velocity = 0

$v(t) = x'(t) = 2t-6$
$0 = 2t-6$
$6 = 2t$
$\boxed{t=3}$

9. Fill in the blanks with correct mathematical notation.

a) If you want the average acceleration of a particle on the interval [1, 3], you must find $\frac{v(3)-v(1)}{3-1}$.

b) If you want the acceleration of a particle at $t = 8$, you must find $a(8) = v'(8)$.

10. Rocket A has a positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds as shown in the table below.

t (sec)	0	10	20	30	40	50	60	70	80
$v(t)$ (ft/sec)	5	14	22	29	35	40	44	47	49

a) Find the average acceleration of Rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

avg. acceler = slope of velocity $= \frac{v(80)-v(0)}{80-0} = \frac{49-5}{80-0} = \frac{44}{80}$ ft/sec/sec

b) Using the data in the table, find an estimate for $v'(35)$. Indicate units of measure.

$\frac{35-29}{40-30} = \frac{6}{10} = \frac{3}{5}$ ft/sec^2

11. A particle moves along the x–axis so that its position at any time $t \ge 0$ is given by the function $x(t) = t^3 - 12t + 1$, where x is measured in feet and t is measured in seconds. Justify each response and indicate units of measure when appropriate.

a) Find the displacement during the first 3 seconds.

$S(3) - S(0) = -8 - 1$

$= -9$

(nine units to left of where it began)

b) Find the average velocity during the first 3 seconds.

Slope of $x(t)$

$\frac{S(3)-S(0)}{3-0} = \frac{-8-1}{3-0} = \frac{-9}{3} = -3 \frac{\text{ft}}{\text{sec}}$

c) Find the instantaneous velocity at $t = 3$ seconds.

$v(t) = s'(t) = 3t^2 - 12$

$v(3) = s'(3) = 3(3)^2 - 12$

$= 27 - 12 = 15$ ft/sec.

d) Find the acceleration when $t = 3$ seconds.

$a(t) = v'(t) = s''(t) = 6t$

$a(3) = v'(t) = s''(3) = 6(3)$

$= 18$ ft/sec^2

e) When is the particle moving left?

When $v(t) < 0$

$3t^2 - 12 < 0$

$3t^2 < 12$

$t^2 < 4$ but $t \ge 0$

$t < |2|$

$-2 < t < 2$

so $\boxed{0 \le t < 2}$

f) At what value(s) t does the particle change direction?

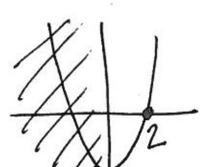
When $v(t)$ changes sign, the particle changes direction

$v(t) = 0$ $\quad 3t^2 - 12 = 0$

$3t^2 = 12$

$t^2 = 4$

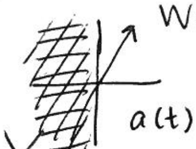
$t = \pm 2$



at $t = 2$ the veloc. changes from − to +, so particle changes direction

g) When is the particle speeding up?

When $v(t)$ & $a(t)$ have same sign.



they have same sign on $(2, \infty)$

13. [Calculator] The cost involved in maintaining annual inventory for a certain manufacturer is given by $C(x) = \frac{1{,}008{,}000}{x} + 6.3x$, where x is the number of items stored. Find the marginal cost of storing the 351$^{\text{st}}$ item.

~~14~~. [Calculator] Suppose that the dollar cost of producing x washing machines is $c(x) = 2000 + 100x - 0.1x^2$.

a) Find the marginal cost when 100 washing machines are produced.

b) Show that the marginal cost when 100 washing machines are produced (your answer from part b) is approximately the cost of producing one more washing machine after the first 100 have been made, by calculating the latter cost directly.

~~15~~. [Calculator] Suppose the weekly revenue (\$) from selling x custom-made office desks is $r(x) = 2000\left(1 - \frac{1}{x+1}\right)$.

Find the marginal revenue when a 6th desk is created.

16. a) Write the area A of a circle as a function of the circumference C.

$A = \pi r^2 \qquad C = 2\pi r \qquad \frac{C}{2\pi} = r$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \pi \cdot \frac{C^2}{4\pi^2} = \boxed{A = \frac{C^2}{4\pi}}$$

b) Evaluate the rate of change of A at $C = 4\pi$.

$$\frac{dA}{dt} = \frac{2C}{4\pi} = \frac{C}{2\pi}\bigg|_{4\pi} = \frac{4\pi}{2\pi} = \boxed{2 \text{ units}^2/\text{unit}}$$

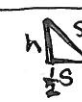
c) If C is measured in miles and A is measured in square miles, what units are used for $\frac{dA}{dC}$?

$\frac{dA}{dC} = $ miles^2/miles   don't reduce label, it no longer makes sense.

17. a) Write the area A of an equilateral triangle as a function of the side length s.

$A = \frac{1}{2}s \cdot h$

$$A = \frac{1}{2}s \cdot \frac{\sqrt{3}}{2}s = \frac{\sqrt{3}}{4}s^2$$



$\left(\frac{1}{2}s\right)^2 + h^2 = s^2$

$-\frac{1}{4}s^2 \quad -\frac{1}{4}s^2$

$\sqrt{h^2} = \sqrt{\frac{3}{4}s^2}$

$h = \frac{\sqrt{3}}{2}s$

b) Find $\left.\frac{dA}{dr}\right|_{s=12}$

$$dA = 2 \cdot \frac{\sqrt{3}}{4}s = \frac{\sqrt{3}}{2}s\bigg|_{12} = \frac{\sqrt{3}}{\cancel{2}} \cdot \cancel{12}$$

$$= \sqrt{3} \cdot 6$$

$$= \boxed{6\sqrt{3}}$$

18. The number of gallons of water in a tank m minutes after the tank has started to drain is given by the equation

$$G(m) = 300\left(20 - t^2\right).$$

$6000 - 300t^2$

a) How fast is the water draining at the end of 5 minutes?

$$G'm = -600t\big|_5 = -3000 \text{ gal/min}$$

b) What is the average rate at which the water drains out of the tank during the first 5 minutes?

Slope of gallons $(0, 6000)$ $(5, -1500)$

$$\frac{6000 - -1500}{0 - 5} = \frac{7500}{-5} = -1500 \text{ gal/min}$$

AP Calculus
3.5 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Find the following derivatives … AND MEMORIZE THEM ASAP!

a) $\frac{d}{dx}[\sin x] = \cos x$ b) $\frac{d}{dx}[\cos x] = -\sin x$ c) $\frac{d}{dx}[\tan x] = \sec^2 x$

d) $\frac{d}{dx}[\sec x] = \sec x \tan x$ e) $\frac{d}{dx}[\csc x] = -\csc x \cdot \cot x$ f) $\frac{d}{dx}[\cot x] = -\csc^2 x$

2. Find $\frac{dy}{dx}$ for each of the following:

a) $y = 3 - x - \tan x$

$y' = -1 - \sec^2 x$

b) $y = \underset{u}{x}\,\underset{v}{\csc x}$

$y' = \cancel{x} \cdot -\csc x \cdot \cot x + \csc x(1)$

$= -x \csc x \cot x + \csc x$

or $\csc x(1 - x \cdot \cot x)$

$x^{-1} + \underset{u}{7x^2} \cdot \underset{v}{\sin x}$

c) $y = \frac{1}{x} + 7x^2 \sin x$

$y' = -x^{-2} + 7x^2(\cos x) + \sin x(14x)$

$= -\frac{1}{x^2} + 7x^2 \cdot \cos x + 14x \sin x$

d) $y = \frac{\cot x \leftarrow u}{5 - \cos x \leftarrow v}$

$y' = \frac{(5-\cos x)(-\csc^2 x) - (\cot x)(\sin x)}{(5-\cos x)^2}$

3. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

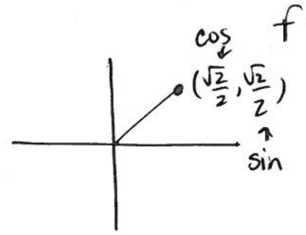
$\sec^2 x - (-\csc^2 x)$

$\boxed{\frac{dy}{dx} = \sec^2 x + \csc^2 x}$

4. If $f(x) = \frac{x}{\tan x}$, then $f'(\frac{\pi}{4}) =$

$f'(x) = \frac{\tan x(1) - x(\sec^2 x)}{(\tan x)^2} \Big|_{\pi/4}$

$f'(\frac{\pi}{4}) = \frac{\tan(\frac{\pi}{4}) - \frac{\pi}{4} \cdot \sec^2(\frac{\pi}{4})}{(\tan\frac{\pi}{4})^2}$



$= \frac{1 - \frac{\pi}{4} \cdot 2}{1^2} = \frac{1 - \frac{\pi}{2}}{1}$

$= \boxed{1 - \frac{\pi}{2}}$

5. If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

$\frac{dy}{dx} = \overset{u}{\sec x}\ \overset{v}{\tan x}$

$\frac{d^2y}{dx^2} = \sec x \sec^2 x + \tan x \cdot \sec x \tan x$

$= \sec^3 x + \tan^2 x \sec x$

~~6.~~ If $y = \overset{u}{\theta}\ \overset{v}{\tan\theta}$, find y''.

$y' = \theta \cdot \sec^2\theta + \tan\theta \cdot (1)$

$= \underbrace{\theta \cdot \sec\theta}_{u} \cdot \underbrace{\sec\theta}_{v} + \tan\theta$

$y'' = \theta \cdot \sec\theta \cdot \sec\theta\tan\theta + \sec\theta(\theta \cdot \sec\theta\tan\theta + \sec\theta) + \sec^2\theta$

7. If $f(x) = \sin x$, find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$. What do you think the function $f^{(100)}(x)$ is?

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$f^{(100)}(x) = \sin x$ since 100 is a multiple of 4.

8. Find an equation of the tangent line and the normal line to the graph of $y = x + \cos x$ at the point (0, 1).

$x = 0$

$y = 1$

$m = 1$

$y' = 1 - \sin x \big|_0$

$= 1 - 0$

$= 1$

$y - 1 = 1(x - 0)$ tangent

$y - 1 = -1(x - 0)$ normal

9. [Calculator] Find equations for the lines that are tangent and normal to the curve $y = x^3 \sin x$ when $x = 2$.

radian mode!

$x = 2$

$y = 8 \cdot \sin(2) = 7.2744$

$m = 7.5824$

$y = x^3 \cdot \sin x$

$y' = x^3 \cdot \cos x + \sin x\,(3x^2)\big|_2$

$= 2^3 \cdot \cos(2) + \sin(2)(12)$

$= \ldots 7.5824$

or nDeriv

$y - 7.2744 = 7.5824(x-2)$ tangent

$y - 7.2744 = -\frac{1}{7.5824}(x-2)$ normal

10. Find the points on the curve $y = \cot x$, $0 < x < \pi$, where the tangent line is parallel to the line $y = -2x$.

where is slope $= -2$?

$y = \cot x$

$y' = -\csc^2 x$

$-\csc^2 x = -2$

$\sqrt{\csc^2 x} = \sqrt{2}$

$\csc x = \pm\sqrt{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

what angle in $0 < x < \pi$ makes $\sin x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$

11. $\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} =$

definition of derivative of $y = \sin x$

A) 0

B) 1

C) $\sin x$

D) $\cos x$

E) nonexistent

~~12.~~ [Calculator] A particle moves along a line so that at time t, $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

~~13.~~ [No Calculator] A spring is bobbing up and down. Its position at any time $t \ge 0$ is given by $s(t) = -4\sin t$.

a) What is the initial position of the spring?

b) Which way is the particle moving to start? Justify your response.

c) At $t = \frac{5\pi}{4}$, is the spring moving up or down? Justify your response.

d) Is the spring speeding up or slowing down at $t = \frac{5\pi}{4}$? Justify your response.

14. [Calculator Required] A body is moving in simple harmonic motion (up/down) with position $s(t) = 3 + \cos t$, where $0 \le t < 2\pi$.

a) Find $v(t)$, the velocity function.

b) Find the zeros of $v(t)$.

c) Find $a(t)$, the acceleration function.

d) Find the zeros of $a(t)$.

e) When is the object stopped? Justify your response.

f) When does the object change direction? Justify your response.

g) When does the object speed up? Justify your response.

15. Suppose $h(x)=\begin{cases}\cos x & x<0\\ x+p & x\geq 0\end{cases}$.

a) Is there a value of p so that $h(x)$ is continuous at $x = 0$? Explain.

$\cos(0) = 1$

$x+p = 1$ $\qquad p = 1$

$0+p = 1$

b) Is there a value of p so that $h(x)$ is differentiable at $x = 0$? Explain.

16. Suppose $y = \tan x$.

a) Graph one period of the function on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

b) Find $\left.\frac{dy}{dx}\right|_{x=0}$. Explain what you have found.

c) Does your answer in part (b) match the graph you made in part (a)? If not, fix it.