AP Calculus AB
Chapter 2 Review

Name: ____________________

Block: ______

**1. (No Calculator) Evaluate each limit or explain why the limit does not exist.**

Try substitution first (don't forget)

b) $\lim_{x\to\infty}\frac{x^2+5x-3}{3x+2}$ $\boxed{\text{DNE} -}$ grows w/o bound

end behav

c) $\lim_{x\to\infty}\frac{x^2+5x-3}{3x^2+2} = \boxed{\frac{1}{3}}$

d) $\lim_{x\to-2}\left(x^3-2x^2+1\right)$

$=(-2)^3-2(-2)^2+1$

$=-8-8+1=\boxed{-15}$

e) $\lim_{x\to\infty}\frac{x^2+5x-3}{3x^3+2} = \boxed{0}$

f) $\lim_{x\to0}\frac{x}{\sin(2x)}\cdot\frac{2x}{2x} = \lim_{x\to0}\frac{\cancel{x}}{\frac{\sin(2x)}{2x}\cdot 2\cancel{x}}$

$=\lim_{x\to0}\frac{1}{2\,\frac{\sin 2x}{2x}}$

$=\frac{1}{2\cdot1}=\boxed{\frac{1}{2}}$

g) $\lim_{x\to\infty}\frac{\sin x}{2x} = \boxed{0}$

end behav.

i) $\lim_{x\to0} e^x\sin x$

$=e^0\cdot\sin(0)$

$=1\cdot0=\boxed{0}$

j) $\lim_{x\to1}\frac{4x^2+5x}{x-3}$ $\frac{4(1)^2+5(1)}{1-3} = \boxed{\frac{-9}{2}}$

l) $\lim_{x\to\infty}\frac{5x-7x^2}{4x^2+1} = \boxed{-\frac{7}{4}}$

m) $\lim_{x\to-3}\frac{|x+3|}{x+3}$ DNE

$\lim_{x\to-3^-}\frac{|x+3|}{x+3}=-1$ $\lim_{x\to-3^+}\frac{|x+3|}{x+3}=1$

n) $\lim_{x\to\infty}\frac{x^4+x^3}{12x^3+128}$ $\boxed{\text{DNE}}$ grows w/o bound

o) $\lim_{x\to4}\sqrt{1-2x}$ $\boxed{\text{DNE}}$

$1-2(4)$

$-7$

can only have domain $x\le\frac{1}{2}$

p) $\lim_{x\to0}\frac{\frac{5}{5}\cdot\frac{1}{x+5}-\frac{1}{5}\cdot\frac{x+5}{x+5}}{x}$

$=\lim_{x\to0}\frac{\cancel{5}}{5(x+5)}-\frac{(x+\cancel{5})}{5(x+5)}\cdot\frac{1}{x}$

$=\lim_{x\to0}\frac{-\cancel{x}}{5(x+5)}\cdot\frac{1}{\cancel{x}} = \lim_{x\to0}\frac{-1}{5(x+5)}$

$=\frac{-1}{5(+5)} = \boxed{\frac{-1}{25}}$

q) $\lim_{x\to9}\frac{\sqrt{x}-3}{x-9}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}$

$\lim_{x\to9}\frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$

$\lim_{x\to9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}$

$=\boxed{\frac{1}{6}}$

r) $\lim_{x\to2}\frac{x^2-x-2}{x^2+x-6}$

$\lim_{x\to2}\frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x+3)}$

$\lim_{x\to2}\frac{x+1}{x+3}=\boxed{\frac{3}{5}}$

3. (Calculator) Make a table of values (4 of them would work) to evaluate $\lim_{x\to2^+}\frac{x+3}{x-2}$.

approaching 2 from the right

| X | $\frac{x+3}{x-2}$ |
|---|---|
| 2.1 | 51 |
| 2.01 | 501 |
| 2.001 | 5001 |
| 2.0001 | 50001 |

$\lim_{x\to2^+}\frac{x+3}{x-2}$ DNE $(\infty)$ (grows w/o bound)

4. (No Calculator) Suppose $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, and

$$\lim_{x\to c}[f(x)+g(x)]=2$$
$$\lim_{x\to c}[f(x)-g(x)]=1$$

Find $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$.

$\lim_{x\to c} f(x) + \lim_{x\to c} g(x) = 2$

$+ \quad \lim_{x\to c} f(x) - \lim_{x\to c} g(x) = 1$

$2\cdot\lim_{x\to c} f(x) = 3$

$\boxed{\lim_{x\to c} f(x) = \frac{3}{2}}$

$\boxed{\lim_{x\to c} g(x) = \frac{1}{2}}$

5. (No Calculator) Find all asymptotes for each function and justify your response.

b) $f(x)=\frac{(\cancel{x+2})(x-3)}{(\cancel{x+2})(x-1)}$

$\lim_{x\to\infty} \frac{x-3}{x-1} = 1$

$\boxed{\text{hole @ } x=-2 \quad \text{VA @ } x=1 \quad \text{HA } y=1}$

c) $f(x)=\frac{x-1}{x^2(x+2)}$

$\boxed{\text{VA } x=0,\ x=-2 \quad \text{HA } y=0}$

end behav.

d) $f(x)=\frac{x^3-3x^2+x-1}{x^2+x-2}$

$(x+2)(x-1)$

$\boxed{\text{VA @ } x=-2,\ x=1 \quad \text{No HA}}$

6. (No Calculator) Let $y=\frac{x^2+5x-3}{x-2}$.

a) Find the End Behavior Model $\frac{x^2}{x} = \boxed{x}$

b) Describe the End Behavior using limits.

$\lim_{x\to\infty} \frac{x^2+5x-3}{x-2}$ $(\infty)$ DNE ($y=x$ grows w/o bound)

$\lim_{x\to-\infty} \frac{x^2+5x-3}{x-2} = \text{DNE } (-\infty)$ ($y=x$ decreases w/o bound)

c) Find all asymptotes.

VA @ $x=2$

Slant asymptote

$$\begin{array}{r|l} & x+7 \\ \hline x-2 & x^2+5x-3 \\ & -x^2+2x \downarrow \\ \hline & 7x-3 \\ & -7x+14 \\ \hline & 9 \end{array}$$

$y=x+7$ is slant asymp

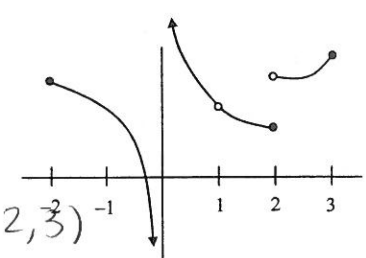
8. Let $h(x)=\frac{(x-1)(x+3)}{(x+3)(x-2)}$. Identify all values of $c$ where the $\lim_{x\to c} h(x)$ EXISTS.

all values of $c \neq 2$

hole @ $x=-3$

so limit exists $x\to-3$

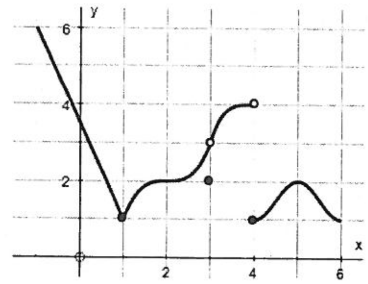
9. The function shown to the right is defined on [–2, 3].
For what values of $c$, does $\lim_{x\to c} f(x)$ exist?




for all values

$[-2,0)\cup(0,2)\cup(2,3)$

**For questions 10 - 13, use the function shown to the right. The domain is [–1, 6].**




10. Evaluate each of the following limits. If they do not exist, explain why.

a) $\lim_{x\to 1} f(x) = 1$

b) $\lim_{x\to 3^-} f(x) = 3$

c) $\lim_{x\to 3^+} f(x) = 3$

d) $\lim_{x\to 3} f(x) = 3$

e) $\lim_{x\to 4^+} f(x) = 1$

f) $\lim_{x\to 4^-} f(x) = 4$

g) $\lim_{x\to 4} f(x)$ DNE
LHL ≠ RHL

h) $\lim_{x\to 0} f(x) = 3.5$

11. For what values of $x$ is the function continuous?

$[-1,3) \cup (3,4) \cup (4,6)$

12. For what values of $x$ is the function not continuous?

$x = 3 \quad x = 4$

13. Are any of the values you used to answer question 11 removable? If so, describe how you would make the function continuous at that point?

removable @ $x = 3$ make $f(3) = 3$

14. Let $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ -x+1 & \text{if } -1 < x < 0 \\ 2 & \text{if } x = 0 \\ -x+1 & \text{if } 0 < x < 1 \\ 2 & \text{if } x \ge 1 \end{cases}$.

a) Find the right-hand and left-hand limits of $f$ at $x = -1$, 0, and 1.

$\lim_{x\to -1^-} f(x) = 2$ $\lim_{x\to -1^+} f(x) = 0$

$\lim_{x\to 0^-} f(x) = 1$ $\lim_{x\to 0^+} f(x) = 1$

$\lim_{x\to 1^-} (f(x)) = 0$ $\lim_{x\to 1^+} f(x) = 2$

b) Does $f$ have a limit as $x$ approaches –1? 0? 1? If so, what is it? If not, why not?

$\lim_{x\to 0} f(x) = 1$ no $\lim_{x\to -1} f(x)$ or $\lim_{x\to 1} f(x)$

LHL ≠ RHL

c) Is $f$ continuous at $x = -1$? 0? 1? Explain.

not continuous @ –1 (jump disc)

not continuous @ 0 (hole)

not continuous @ 1 (jump disc)

15. Find the value of the parameter that would make each function continuous.
Justify your response *using the definition of continuity*.

a) $j(x)=\begin{cases} ax^2 & ;x<1 \\ 4x-2 & ;x\geq 1 \end{cases}$  $a=2$

$4(1)-2=2$
$ax^2=2$
$a(1)^2=2$
$a=2$

left Hand limit = right hand limit

b) $k(x)=\begin{cases} \frac{\sin 3x}{x} & x\neq 0 \\ a & x=0 \end{cases}$  $\lim_{x\to 0} \frac{\sin 3x}{x}\cdot\frac{3}{3}=3$

$a=3$

$\lim_{x\to 0} k(x)=3$ so $k(0)=3$

c) $f(x)=\begin{cases} \frac{x^2-2x}{x} & \text{if } x\neq 0 \\ b & \text{if } x=0 \end{cases}$  $\lim_{x\to 0} \frac{\cancel{x}(x-2)}{\cancel{x}}=\lim_{x\to 0}(x-2)=-2$

$b=-2$

$\lim_{x\to 0} f(x)=-2$ so $f(0)=-2$

16. Let $k(x)=\frac{\sqrt{x}-3}{x-9}$. Write an extension to the function so that it is continuous at $x=9$.

$\lim_{x\to 9} k(x)=\frac{1}{6}$ (from #18)

$K(x)=\begin{cases} \frac{\sqrt{x}-3}{x-9}, & x\neq 9 \\ \frac{1}{6}, & x=9 \end{cases}$

17. [No Calculator] Find the average rate of change of $f(x)=3-\sin x$ over the interval $\left[0,\frac{\pi}{2}\right]$.

SLOPE

19. Let $g(x)=\sqrt{x}$. Find the instantaneous slope at $x=4$.

20. Let $y = x^3 - 4x$.

a) Find the instantaneous slope for any value of $x = a$.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(a+h)^3 - 4(a+h) - (a^3 - 4a)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{a^3} + 3a^2h + 3ah^2 + h^3 - \cancel{4a} - 4h - \cancel{a^3} + \cancel{4a}}{h} = \lim_{h \to 0} \frac{\cancel{h}(3a^2 + 3a\overset{0}{\cancel{h}} + \overset{0}{\cancel{h^2}} - 4)}{\cancel{h}}$$

$$= \boxed{3a^2 + -4}$$

b) Use your answer in part $a$ to find the slope at $x = -1$.

$3(-1)^2 - 4 = 3 - 4 = -1$

c) Find the equation of the tangent line when $x = -1$.

$x = -1 \quad y = 3 \quad m = -1 \qquad y - 3 = -1(x+1)$

d) Find the equation of the normal line when $x = -1$.

$x = -1 \quad y = 3 \quad m = 1 \qquad y - 3 = 1(x+1)$

21. Let $h(x) = x^2 + 3x - 1$.

a) Find the slope of the curve at $x = 1$.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(a+h)^2 + 3(a+h) - 1 - (a^2 + 3a - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 + \cancel{3a} + 3h - \cancel{1} - \cancel{a^2} - \cancel{3a} + \cancel{1}}{h} = \lim_{h \to 0} \frac{\cancel{h}(2a + \overset{0}{\cancel{h}} + 3)}{\cancel{h}} = \boxed{2a + 3}$$

b) Find the equation of the tangent line at the point when $x = 1$.

$x = 1$
$y = 3 \quad m = 5 \qquad y - 3 = 5(x - 1)$

c) Find the equation of the normal line at the point when $x = 1$.

$x = 1$
$y = 3 \quad m = -1/5 \qquad y - 3 = -1/5(x - 1)$

d) At what point(s), if any, are the tangents to the graph of $h(x)$ horizontal? … [Use Calculus!]

$2a + 3 = 0$ (slope of tangent line $= 0$)

$2a = -3$

$$\boxed{a = -\frac{3}{2}}$$